

An Efficient New Sequential Strategy for Multi-Objective Optimization using LS-OPT[®]

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Abstract

A new surrogate-assisted Multi-Objective Optimization algorithm has been implemented in LS-OPT[®]. The algorithm, known as Pareto Domain Reduction, is an adaptive sampling method and an extension of the classical Domain Reduction approach (also known as SRSM). A Multidisciplinary Design Optimization (MDO) example involving a vehicle impact is used to demonstrate that the accuracy is very close to the NSGA-II “exact” method while using a small fraction of the computational effort.

Introduction

Historically, two methods have been available for the solution of Multi-objective optimization problems using LS-OPT[®] [1]. The first is a direct optimization method: the NSGA-II as developed by Deb [2]. The second approach reduces the number of simulations by using a metamodel (typically a Radial Basis Function Network or RBF) to approximate the true design problem. The NSGA-II algorithm can be used to obtain “exact” results but requires a very large number of simulations so can be very expensive, especially in crashworthiness optimization where simulations involve highly detailed Finite Element models and nonlinear dynamic analysis.

The surrogate-assisted method for MOO problems is the faster of the two methods but currently relies on uniform global sampling to construct the metamodel. This implies that during an iterative solution, the global accuracy of the metamodel is gradually improved at the expense of local accuracy in the neighborhood of the solution, an effect that delays convergence. For single objective problems, this deficiency is addressed in LS-OPT by the Domain Reduction approach. This method has been available in LS-OPT since its inception (known as Sequential Response Surface Method or SRSM [1,3]), but has not been available for multi-objective problems in which multiple solutions are possible. Surrogate-assisted Multi-objective optimization algorithms have been extensively described in literature, see e.g. Syberfeldt *et al* [4].

In this study a new adaptive domain reduction method known as Pareto Domain Reduction (PDR) is introduced for improving efficiency and accuracy. The method employs heuristics which are similar to SRSM but since multiple optimal solutions are possible, it uses the irregular subregion of the Pareto Optimal Frontier (POF) as a sampling domain. The size of this subregion is iteratively reduced in order to intensify the exploration the neighborhood of the POF. The method is conservative in the sense that early sampling is global with a gradual convergence to the POF. Hence it can also be viewed as an adaptive sampling approach. The method has the additional advantage that, if a multi-objective problem is posed so that it has only one optimal solution, it degenerates to SRSM which unifies and simplifies LS-OPT methodology and user choice.

Since the RBF is the default metamodel in LS-OPT, a Space Filling approach is used to obtain a well spaced sampling in the reduced domain. This avoids point duplication and maximizes the accuracy in the POF neighborhood.

During the development phase of the new method, several other features were added to facilitate and automate an optimization run. The most important of these allows a selected number of computed POF points (verification runs) to be solved during the final iteration. This step is required to produce the final set of trade-off points from which the user can select a suitable design. These points are based on actual simulations, so the resulting trade-off curve can be used directly for design, without any further simulations.

The remainder of the paper deals with a brief description of the methodology followed by a Multi-objective MDO crashworthiness/modal analysis example of a vehicle. The model is not very large, but sufficiently realistic and representative of a typical model used in industry. The results show that the PDR method is highly accurate and potentially an order of magnitude cheaper than a direct approach.

Pareto Domain Reduction Methodology

The algorithm is based on a simple concept consisting of the following major steps:

Initial conditions:

1. $k := 1$, choose m simulation designs in the full design space.
2. Conduct the simulations, build the surrogate model and construct an approximate POF.

For each iteration k :

1. Select m kernels from the POF.
2. Adapt the subregion size based on iteration ($k-1$)
3. Center a subregion on each kernel.
4. Populate each subdomain with a number of *diversity* design points.
5. Select m points for simulation from the diversity design points.
6. $k := k + 1$
7. Conduct m simulations, build the surrogate model and construct an approximate POF.

Figure 1 shows a typical sampling adaptation to the Pareto Domain. In this case, 5 simulation points are added per iteration (which is the default for 2 variables). The red points represent the most recent simulations in the progression.

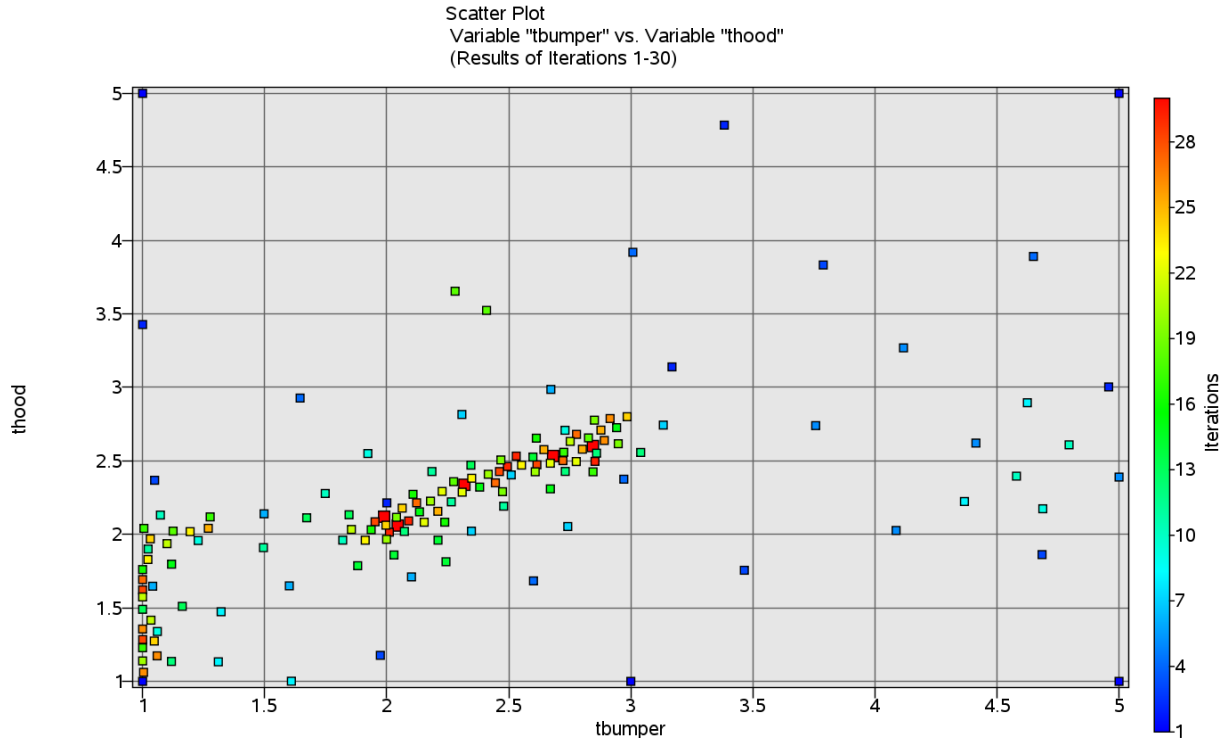


Figure 1: Typical sampling pattern for the Pareto Domain Reduction method (two variable problem).

Example

The crashworthiness simulation considers a model containing approximately 30,000 elements of a National Highway Transportation and Safety Association (NHTSA) vehicle undergoing a full frontal impact. A modal analysis is performed on a so-called ‘body-in-white’ model containing approximately 18,000 elements. The crash model for the full vehicle is shown in Figure 2 for the undeformed and deformed (time = 78ms) states, and with only the structural components affected by the design variables, both in the undeformed and deformed (time = 72ms) states, in Figure 3. The NVH model is depicted in Figure 4 in the first torsion vibrational mode. Only body parts that are crucial to the vibrational mode shapes are retained in this model. The design variables are all thicknesses or gages of structural components in the engine compartment of the vehicle (Figure 3), parameterized directly in the LS-DYNA input file. Twelve parts are affected, comprising aprons, rails, shotguns, cradle rails and the cradle cross member (Figure 3). LS-DYNA v.971 is used for both the crash and NVH simulations, in explicit and implicit modes respectively.

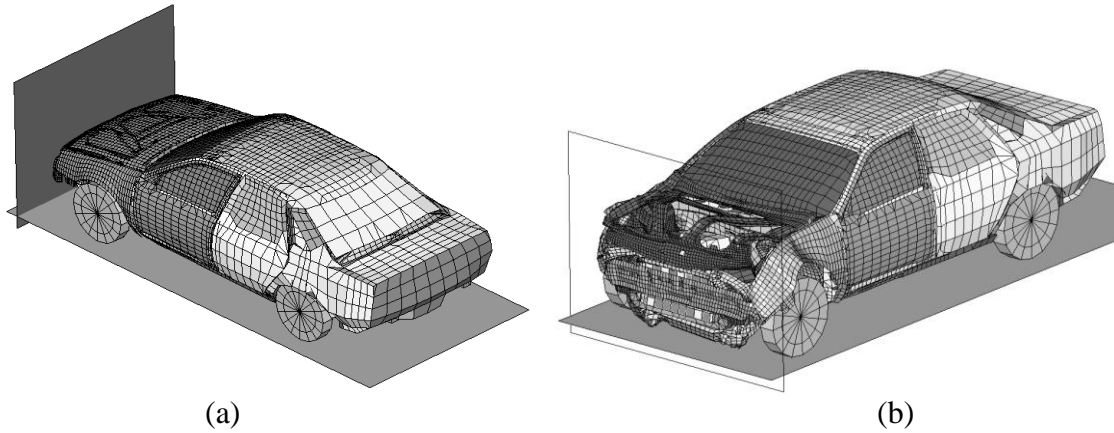


Figure 2: Crash model of vehicle showing road and wall a) Undeformed b) Deformed (78ms)

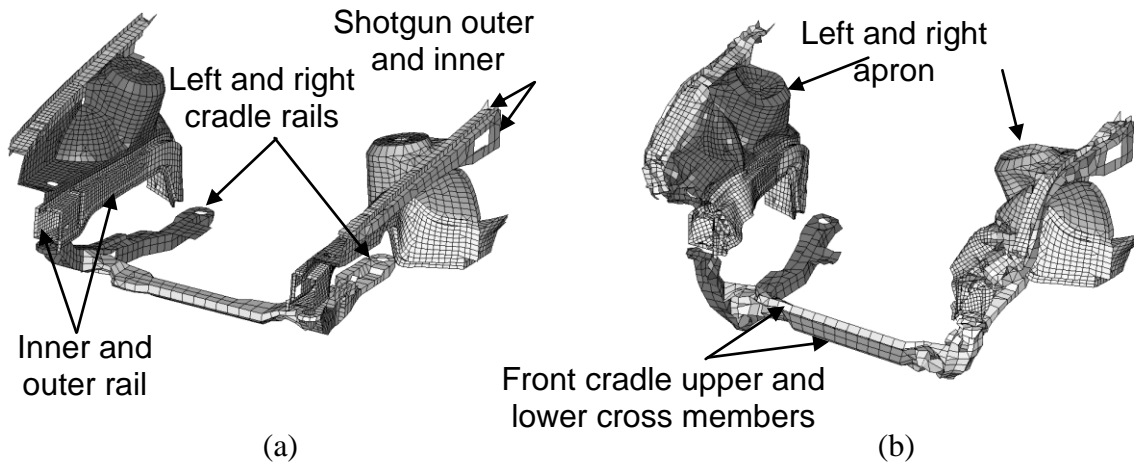


Figure 3: Structural components affected by design variables – a) Undeformed and (b) deformed (time = 72ms)

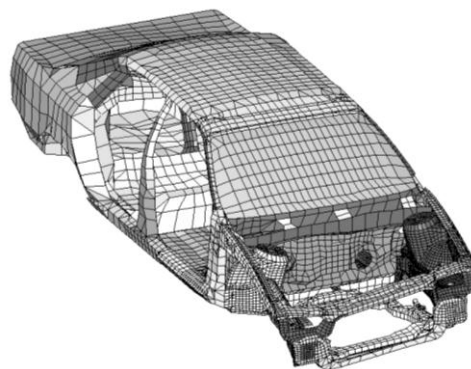


Figure 4: Body-in-white model of vehicle in torsional vibration mode (38.7Hz)

The design formulation is as follows:

$$\begin{aligned}
 & \text{Minimize} && \text{Mass} \\
 & \text{Minimize} && \text{Maximum intrusion} \\
 & \text{subject to} && \\
 & && \text{Maximum intrusion}(\mathbf{x}_{\text{crash}}) < 551.27\text{mm} \\
 & && \text{Stage 1 pulse}(\mathbf{x}_{\text{crash}}) > 14.51\text{g} \\
 & && \text{Stage 2 pulse}(\mathbf{x}_{\text{crash}}) > 17.59\text{g} \\
 & && \text{Stage 3 pulse}(\mathbf{x}_{\text{crash}}) > 20.75\text{g} \\
 & && 41.38\text{Hz} < \text{Torsional mode frequency}(\mathbf{x}_{\text{NVH}}) < 42.38\text{Hz}
 \end{aligned}$$

Variables:

$$\mathbf{x}_{\text{crash}} = \mathbf{x}_{\text{NVH}} = [\text{rail_inner}, \text{rail_outer}, \text{cradle_rails}, \text{aprons}, \text{shotgun_inner}, \text{shotgun_outer}, \text{cradle_crossmember}]^T$$

The three stage pulses are calculated from the SAE filtered (60Hz) acceleration and displacement of a left rear sill node in the following fashion:

$$\text{Stage } i \text{ pulse} = \frac{-k}{d_2 - d_1} \int_{d_1}^{d_2} a dx ;$$

$$k = 0.5 \text{ for } i = 1, 1.0 \text{ otherwise;}$$

with the limits $[d_1; d_2] = [0; 184]; [184; 334]; [334; \text{Max}(\text{displacement})]$ for $i = 1, 2, 3$ respectively, all displacement units in mm and the minus sign to convert acceleration to deceleration. The Stage 1 pulse is represented by a triangle with the peak value being the value used.

Three optimization strategies were used to verify and compare the results:

1. A direct optimization using the NSGA-II algorithm. A tolerance on the hypervolume change of 10^{-4} was used to create a realistic benchmark result.
2. A metamodel-assisted optimization using sequential global Radial Basis Function networks (LS-OPT Version 4.2). A stopping criterion of 30 iterations was selected.
3. A metamodel-assisted optimization using the new Pareto Domain Reduction approach. A stopping criterion of 30 iterations was selected.

The parameters for each run were as shown in Table 1.

Table 1: Run statistics for Example 1. *Note that the number of simulations for the metamodel assisted methods includes a 100 final verification runs to construct a full trade-off curve. This number is user-selectable and determines the size of the trade-off curve.

	Number of iterations/ generations	Number of simulations per iteration	Total number of simulations per case, including 100 verification simulations for the last iteration	Run configuration (Number of concurrent jobs × MPP cores per simulation)
Direct NSGA-II	75	160	12,000	80×4
Global	30	13	390+100*	13×4
PDR	30	13	390+100*	13×4

All runs were conducted on a 768-core Xeon-based cluster running up to 80×4-core MPP-DYNA simulations in parallel. The SunGrid Engine queuing system was used with job monitoring using the LS-OPT GUI.

Results

Because the selection of the final trade-off curve for the metamodel-based methods is based on an approximation, LS-OPT provides a feature for *a priori* selecting a desired number of trade-off points (which can be different from the number of points in each iteration). If this number appears to be insufficient, the curve can be augmented in a simple post-processing run (change the number and re-start). The trade-off points are evenly spread throughout the Pareto domain using a space filling algorithm for optimal representation.

Although it cannot be proven (there is no mathematical criterion for global optimality) the Direct GA run is considered to be “exact” in the sense that it was run to a very fine tolerance (10^{-4}) of the dominated hyper-volume change.

Figure 5 shows a comparison of the trade-off curves produced by the Direct GA optimization and the PDR run. The PDR results are simulation results of the final trade-off curve (i.e. *not* the approximate results). Note that the trade-off curve is discontinuous (two major discontinuities) with several concave and convex sections. In Figure 5, the PDR result can be seen to closely approximate the exact result. The wall clock time for the PDR method was approximately 7 hours for the entire optimization.

Figure 6 shows a comparison of the predicted trade-off curve and the computed curve using the PDR method which confirms the accuracy of the metamodel and sampling strategy. Note that the metamodel is based on all 390 (13×30) simulations, and not only the last 13.

The new PDR method was also compared to the existing global sampling approach (Figure 7) in which the design points for building the metamodel are evenly distributed globally using a Space Filling sampling approach. The benefit of using the PDR method is clearly visible. Note that finer features such as discontinuities are not properly modeled by the global sampling, obviously because of a sparsity of simulation results near the optimum.

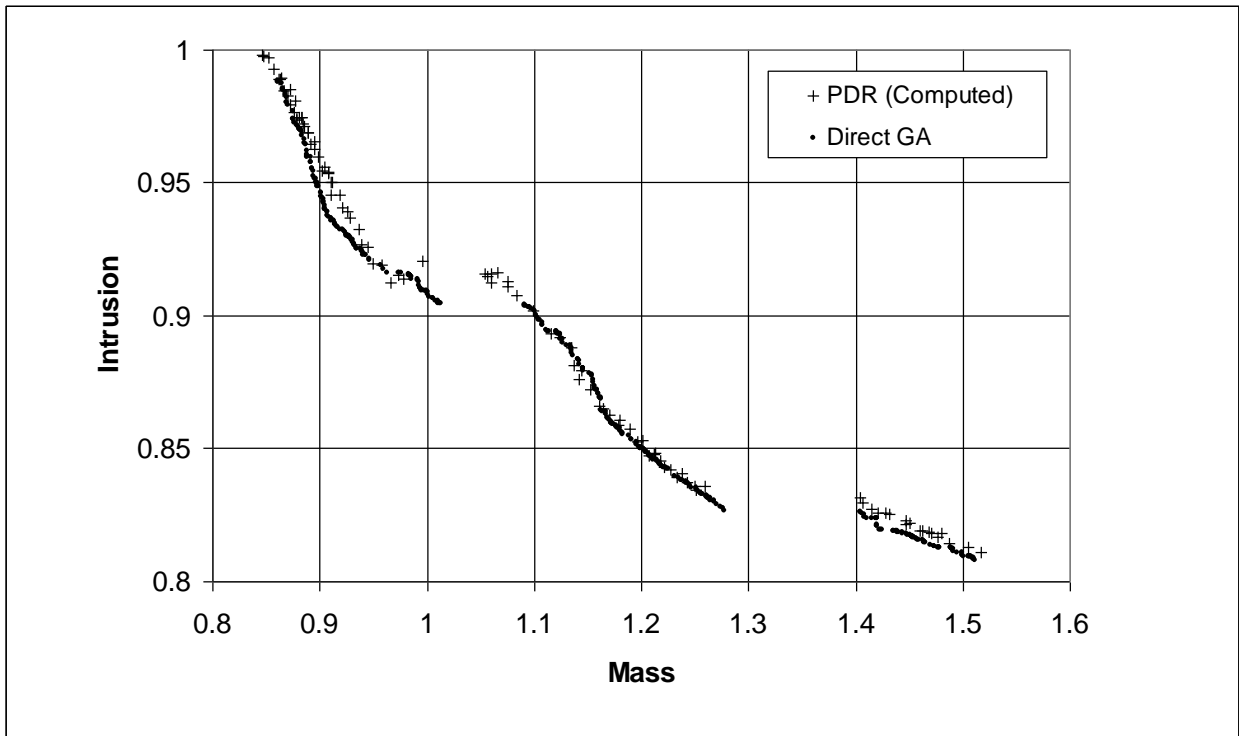


Figure 5: Pareto Optimal Frontier: Comparison of the *computed* POF (100 designs) obtained using the PDR approach with the reference POF computed using the Direct NSGA-II algorithm

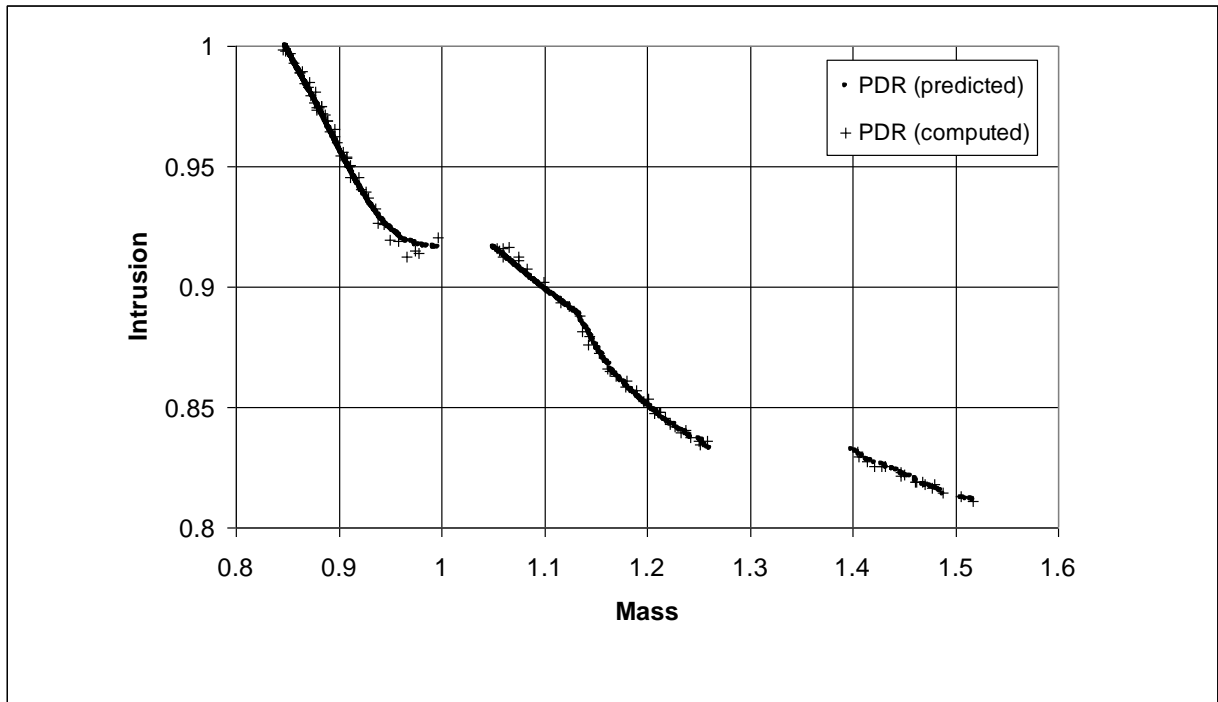


Figure 6: Pareto Optimal Frontier: Comparison of the PDR predicted POF and the PDR computed POF.

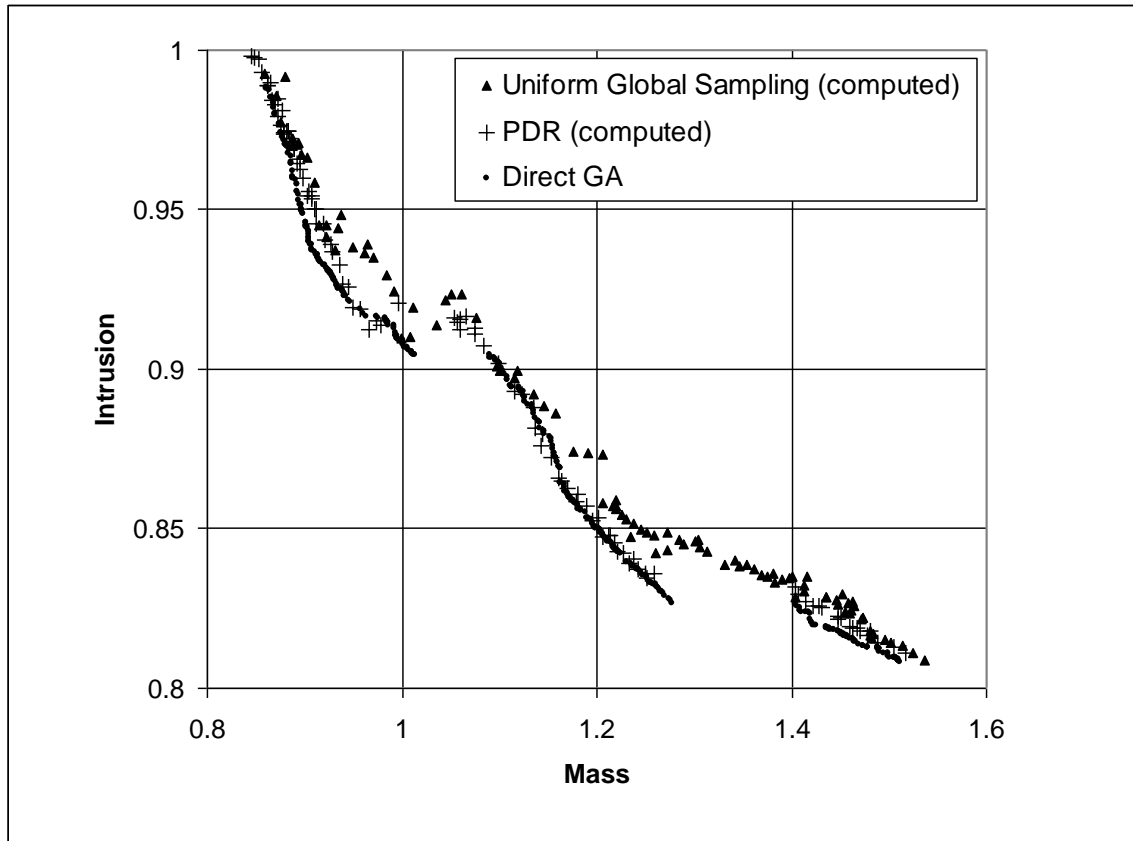


Figure 7: Pareto Optimal Front: Comparison of the simulation results for uniform global sampling (▲) and PDR sampling (+) with the reference solution obtained using the Direct NSGA-II algorithm (●).

Conclusions

The study presents an efficient new surrogate-assisted method for computing trade-off curves using LS-OPT. The new method, known as Pareto Domain Reduction, is almost as accurate as the Direct GA method while using a small fraction of its simulation effort.

Naturally, some MOO examples may only possess single optima, due to the non-conflicting nature of the objective functions involved. In this case, PDR devolves to the classical SRSM available in LS-OPT. This simplifies user choice and means that the SRSM and PDR methods are unified and that PDR therefore applies to a broad class of optimization problems.

References

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4. Syberfeldt, A., Grimm, H., Ng, A. and John, R.I. A Parallel Surrogate-Assisted Multi-Objective Evolutionary Algorithm for Computationally Expensive Optimization Problems, IEEE World Congress on Computational Intelligence, pp. 3177-3184, 1-6 June, 2008.