A Method for the Conversion of Experimental Flow Curves to constant Strain Rates as Input for Material Models

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1 Motivation

To derive material data for finite element simulations, tensile tests are often performed at several constant haul-off speeds. From those tests, flow curves at different strain rates are determined and used as input curves in a material card. Common material models interpolate between those curves to calculate stresses at arbitrary strain rates. In order to do so, a constant strain rate value has to be assigned to each input curve.

However, during tensile tests and corresponding simulations, the strain rate often increases due to strain localization or decreases due to specimen elongation. Therefore, no constant strain rate can be assigned to the flow curves determined from those tests. Instead, the curves need to be modified before they can be used for material modeling. Otherwise the results of a simulation will differ from the experiments.

One possibility to generate flow curves at constant strain rates is to fit a function $\sigma(\varepsilon,\dot{\varepsilon})$ to the measured data and to calculate curves $\sigma_i(\varepsilon)=\sigma(\varepsilon,\dot{\varepsilon}_i)$ at strain rates $\dot{\varepsilon}_i$. However, it is hard to find a function which can be fitted to arbitrary experimental data with satisfying accuracy over the whole range of strains and strain rates. Also the fitting procedure itself is time consuming and can be problematic.

In this paper an algorithm is presented, which can be used to convert experimental flow curves measured at several varying strain rates to curves at arbitrary constant strain rates. It does not involve any curve fitting but gives exact results. The algorithm is based on the inversion of an interpolation matrix by assuming a linear or logarithmic dependence of the stress on the strain rate. Using the converted curves in a material model can considerably improve simulation results. Finally, an outlook on further developments of the algorithm will be given.

2 Conversion Algorithm

This section describes the conversion algorithm, which solves the problem of finding curves at constant strain rates. For ease of use, the algorithm has been implemented with a graphical user interface.
In the following equations, all symbols (stresses, strain rates) denoted with a tilde refer to quantities at constant strain rates, while all symbols without a tilde correspond to the quantities at varying strain rates.

2.1 Interpolation Equations

Material models (as for example **`*MAT_024`** or **`*MAT_SAMP-1`** in LS-DYNA, see [1]), that use several input curves at different constant strain rates to model rate dependent material behavior, calculate stresses at any given strain rate by interpolating between those input curves. This is depicted in Fig.1 for the simple case of only two input curves $\tilde{\sigma}_k$ and $\tilde{\sigma}_{k+1}$ at assigned constant strain rates $\tilde{\dot{\varepsilon}}_k$ and $\tilde{\dot{\varepsilon}}_{k+1}$, respectively.

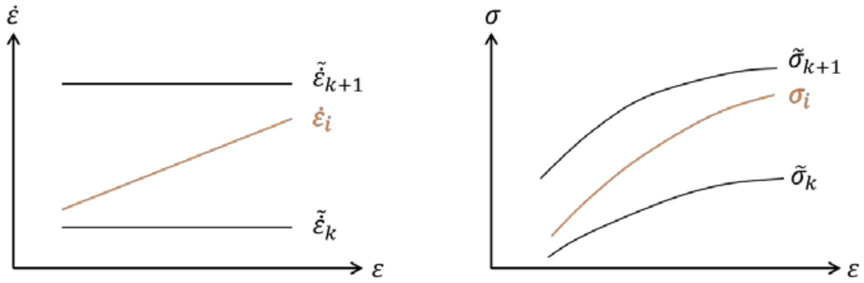

Fig.1: Interpolation of a stress-strain-curve at varying strain rate (shown in red) using two input curves at constant strain rates (shown in black).

For any given strain rate $\dot{\varepsilon}_i$ in between, the corresponding stress σ_i is calculated according to the interpolation equation

$$\sigma_i = (1 - f_i)\tilde{\sigma}_k + f_i \tilde{\sigma}_{k+1} \tag{1}$$

with the interpolation factor

$$0 \le f_i = \frac{\dot{\varepsilon}_i - \tilde{\dot{\varepsilon}}_k}{\tilde{\dot{\varepsilon}}_{k+1} - \tilde{\dot{\varepsilon}}_k} \le 1 \tag{2}$$

which is the normalized distance of $\dot{\varepsilon}_i$ to the nearest lower constant strain rate $\tilde{\dot{\varepsilon}}_k$. The interpolation factor according to equation (2) assumes a linear dependence of the stress on the strain rate. A logarithmic dependence, which is observed for most materials, can also be modeled by using a slightly modified interpolation factor

$$f_i^{\log} = \frac{\ln\left(\dot{\varepsilon}_i / \tilde{\dot{\varepsilon}}_k \right)}{\ln\left(\tilde{\dot{\varepsilon}}_{k+1} / \tilde{\dot{\varepsilon}}_k \right)} . \tag{3}$$

The dependence of all stresses and strain rates from the strain ε was omitted in equations (1)-(3) for the sake of clarity.

Now there are given n different input curves $\tilde{\sigma}_i$ at constant strain rates $\tilde{\dot{\varepsilon}}_i$ ($i = 1,...,n$). To calculate the stresses σ_i at n different varying strain rates $\dot{\varepsilon}_i$, a set of interpolation equations (1) is used, which can also be written in a more compact form as a matrix equation

$$\boldsymbol{\sigma}(\varepsilon) = \mathbf{M}(\varepsilon)\tilde{\boldsymbol{\sigma}}(\varepsilon) \tag{4}$$

with

$\boldsymbol{\sigma}(\varepsilon) = (\sigma_1(\varepsilon),...,\sigma_n(\varepsilon))^T$ and $\tilde{\boldsymbol{\sigma}}(\varepsilon) = (\tilde{\sigma}_1(\varepsilon),...,\tilde{\sigma}_n(\varepsilon))^T$.

If the $n \times n$ interpolation matrix $\mathbf{M}$ is known, its inverse can be determined in order to calculate the stresses $\tilde{\sigma}_i$ at constant strain rates out of any given stresses σ_i at varying strain rates. This solves the problem of finding input curves that can be used for calibrating a material card with experimental data (Fig.2).

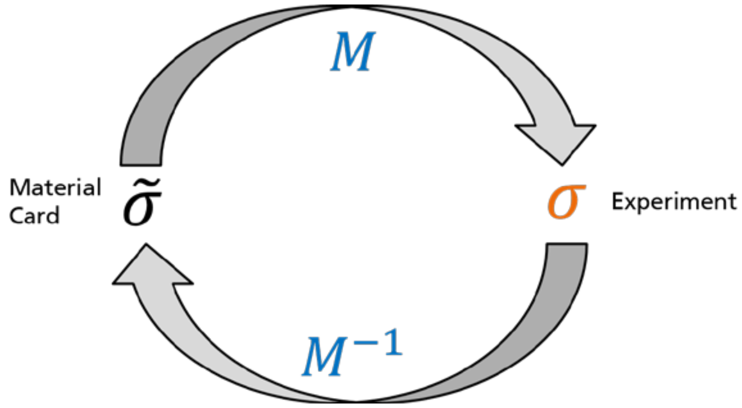

Fig.2: Transformation of stresses between material card and experiment using the interpolation matrix $\mathbf{M}$.

2.2 Interpolation Matrix

The problem of converting input curves to constant strain rates has now been translated into finding an interpolation matrix $\mathbf{M}(\varepsilon)$ and its inverse.

Each line i of this matrix corresponds to one stress $\sigma_i(\varepsilon)$ at strain rate $\dot{\varepsilon}_i(\varepsilon)$ which is calculated according to equation (1) using the two adjacent input curves $\tilde{\sigma}_k(\varepsilon)$ and $\tilde{\sigma}_{k+1}(\varepsilon)$ at constant strain rates $\tilde{\dot{\varepsilon}}_k$ and $\tilde{\dot{\varepsilon}}_{k+1}$, respectively (it is assumed here that the constant strain rates are sorted in ascending order). Therefore, k is the largest value for which $\tilde{\dot{\varepsilon}}_k < \dot{\varepsilon}_i(\varepsilon)$. In other words, the interpolation is done using the two input curves at the nearest lower and higher constant strain rates.

Line i of the interpolation matrix contains two non-zero entries: In column k the matrix element is $M_{ik} = 1 - f_i$ and in column $k+1$ it is $M_{i(k+1)} = f_i$ (cf. equation (1)). All other elements in this line are zero. The interpolation factor f_i is calculated using equation (2) for a linear, or equation (3) for a logarithmic strain rate dependence.

The complete interpolation matrix $\mathbf{M}(\varepsilon)$ at a given strain ε is built up by repeating this procedure for all lines $i = 1,...,n$. In the following, two examples of such a matrix are given which refer to Fig.3, which shows three varying strain rates and three constant strain rate values.

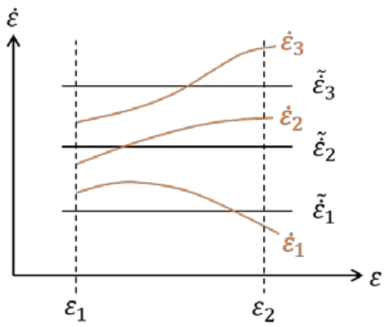

Fig.3: An example showing three varying (red) and three constant (black) strain rates as a function of strain.

At strain $\varepsilon = \varepsilon_1$ the matrix $\mathbf{M}$ has the following form:

$$\mathbf{M}(\varepsilon_1) = \begin{pmatrix} 1-f_1 & f_1 & 0 \\ 1-f_2 & f_2 & 0 \\ 0 & 1-f_3 & f_3 \end{pmatrix}. \qquad (5)$$

At strain $\varepsilon = \varepsilon_2$ two different boundary conditions have to be considered in this example: the first strain rate $\dot{\varepsilon}_1(\varepsilon_2)$ falls below the lowest, and the third strain rate $\dot{\varepsilon}_3(\varepsilon_2)$ rises above the highest constant strain rate value. As no extrapolation is done in LS-DYNA, the corresponding stresses are simply taken from the input curve with the lowest and the highest strain rate value, respectively [1]. Therefore, the corresponding matrix element in $\mathbf{M}$ has to be equal to one:

$$\mathbf{M}(\varepsilon_2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-f_2 & f_2 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (6)$$

2.3 Calculation of Curves at constant Strain Rates

After having determined the interpolation matrix, it is straight forward to calculate the stresses $\tilde{\boldsymbol{\sigma}}$ at constant strain rates:

$$\tilde{\boldsymbol{\sigma}}(\varepsilon) = \mathbf{M}^{-1}(\varepsilon)\boldsymbol{\sigma}(\varepsilon) \,. \tag{7}$$

The inverse matrix $\mathbf{M}^{-1}$ can be determined using standard methods of linear algebra.
However, in some cases an inverse does not exist, which means that the problem has no or no unique solution. This can happen if several strain rates $\dot{\varepsilon}_i$ lie in between the same two constant strain rates while between other constant strain rate values no $\dot{\varepsilon}_i$ occur. This can lead to linearly dependent matrix columns, and in some cases a column which contains only zeros.
As an example, Fig.4 shows the previous three varying strain rates with the constant strain rates $\dot{\varepsilon}_2$ and $\dot{\varepsilon}_3$ shifted to higher values. This results in the following interpolation matrix at strain $\varepsilon = \varepsilon_1$:

$$\mathbf{M}(\varepsilon_1) = \begin{pmatrix} 1-f_1 & f_1 & 0 \\ 1-f_2 & f_2 & 0 \\ 1-f_3 & f_3 & 0 \end{pmatrix} . \tag{8}$$

By crossing out the last column and the corresponding line of the interpolation matrix and inverting the reduced matrix, the remaining stresses $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ can still be calculated.

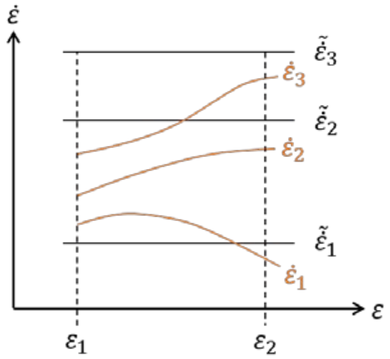

Fig.4: An example of three varying and three constant strain rates for which the inverse matrix does not exist.

3 Results

The presented algorithm was used to determine input curves at constant strain rates for a thermoplastic material. Tensile tests have been performed at three different haul-off speeds (0.5 mm/s, 50 mm/s, and 5000 mm/s). The left diagram in Fig.5 shows the initial input curves as dashed lines, which were calculated directly from the experimental results (together with an upper and lower boundary curve (black lines) which have been included so the material model can extrapolate the stresses at very low or high strain rates). The right diagram shows the constant strain rate values (0.01 /s, 1 /s, and 100 /s) that have been assigned to those input curves as dashed lines. Furthermore, the strain rates that have been measured during the experiments are shown as gray curves. It is obvious that those are not constant. In particular, at strains higher than 0.6, the two lowest strain rates decrease rapidly. Using the input curves shown in Fig.5 in a material model (**`*MAT_024`** with linear strain rate interpolation) for simulating the tensile tests, results in stress-strain- and force-displacement-curves as shown in Fig.6. Clearly, the simulated stresses and forces (dashed lines) fall below the experimental curves as the strain rates decrease at higher strains.

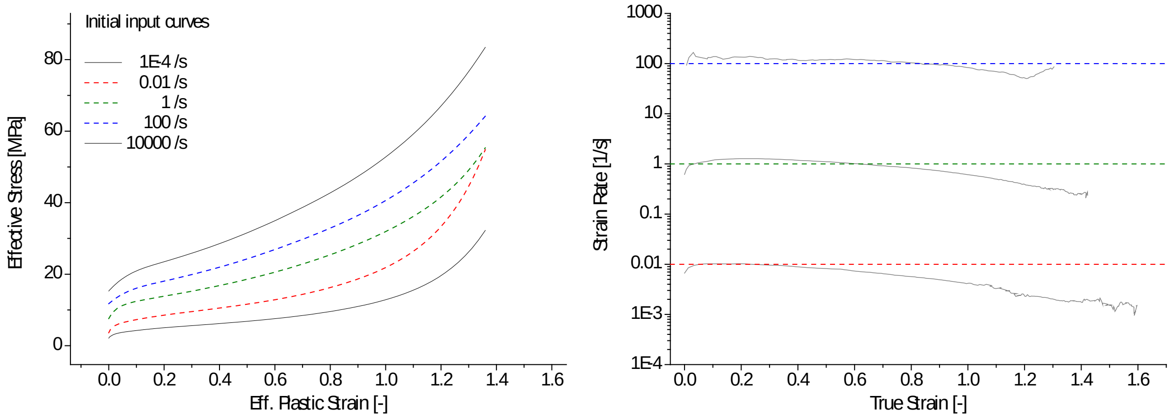

*Fig.5: Left: Initial input curves calculated from tensile test data.
Right: Strain rates during the tests (gray lines) and assigned constant strain rates of the input curves (dashed lines).*

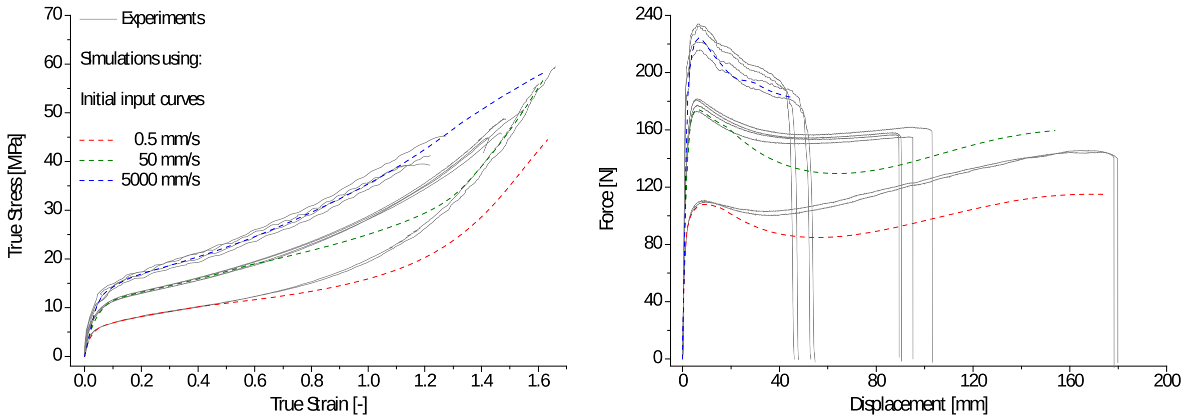

Fig.6: Simulation results using the initial input curves compared to the experimental data.

The converted input curves at constant strain rates for this material are shown as solid lines in the left diagram in Fig.7. There are only small differences compared to the initial curves (dashed lines). However, the constant strain rates to which those curves have been converted are considerably different (solid lines in the right diagram in Fig.7). In fact, they lie exactly in between the strain rates that have been measured, which ensures stable material behavior. If during the simulations the strain rate crossed a constant strain rate value that had been assigned to one of the input curves, this would lead to a sudden change in strain rate hardening which would trigger a strain localization. This behavior was not observed during the tensile tests, but it is a result of the linear strain rate interpolation that has been used in the material model, whereas the material behavior shows a logarithmic dependence. By converting the input curves to constant strain rates in between the experimental rates, such a crossing can be avoided. This approach is useful for material models that only support linear strain rate interpolation.
The results of the simulations using the converted input curves are finally shown in Fig.8. The simulated stresses lie exactly on the measured curves. Also the forces show a much better agreement with the experiments.

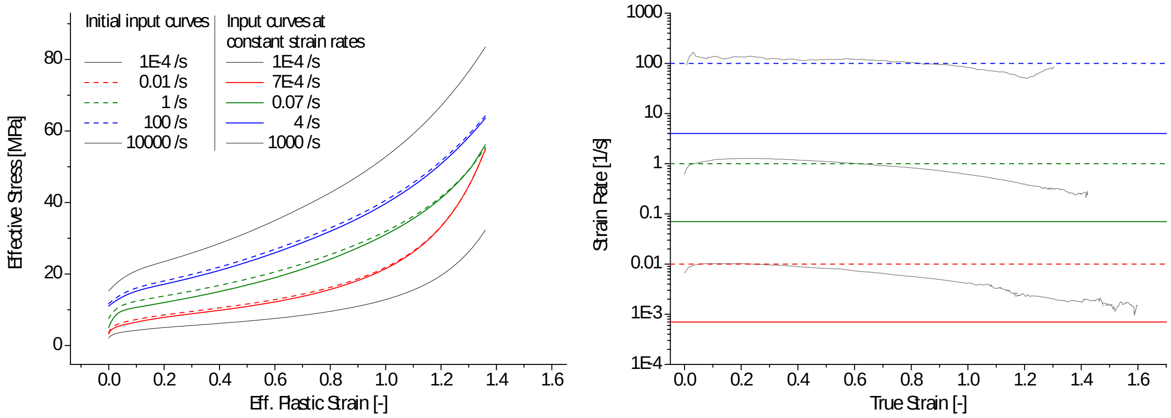

Fig.7: Left: Initial and converted input curves.
Right: Strain rates during the tests (gray lines) and assigned constant strain rates of the initial (dashed lines) and converted (solid lines) input curves.

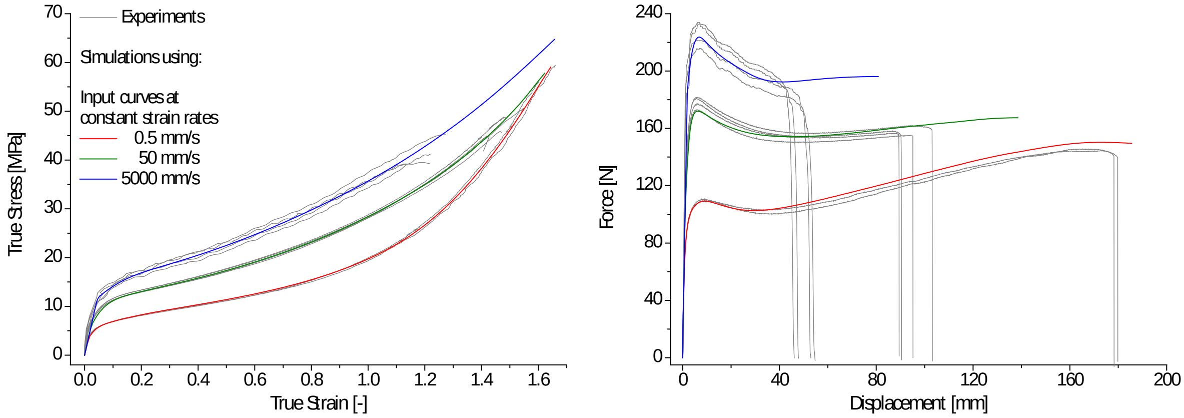

Fig.8: Simulation results using the input curves at constant strain rates compared to the experimental data.

4 Further Developments

The algorithm introduced in the previous sections is useful to determine input curves for material models as for example **`*MAT_024`**. However, the calculated curves are not correct in a physical sense as the interpolation equation (1) assumes an instantaneous reaction of the material to a change in strain rate. But especially polymers are known for their viscous properties which means that they need a certain time to react to a sudden change in the strain rate [2]. This is illustrated in Fig.9. After the strain rate jump, the stress will not follow the dashed line, but it will slowly approach the higher stress values instead. Therefore, the stress-strain-curves measured in tensile tests at constant strain rates (instead of constant haul-off speeds) will be different from the curves calculated using the algorithm, in particular if the duration of the tests is comparable to or less than the relaxation time.

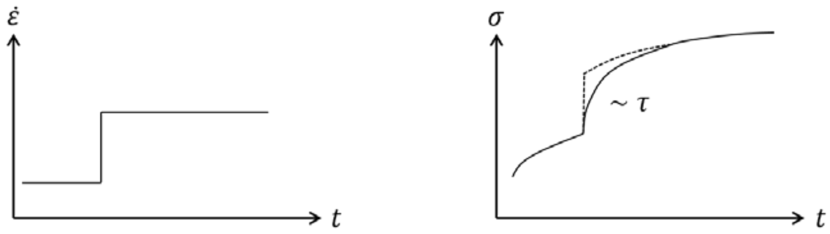

Fig.9: Instantaneous and delayed material response to a sudden change in strain rate.

In order to incorporate those viscous effects into the algorithm, a rheological model is used which is shown in Fig.10. It consists of two parts connected in parallel. The first one contains a “black box”-element which models the quasi-static material behavior. No details are necessary about this element, as the rheological model is not intended to be used as a material model in the classical sense, but it only motivates the equations for the enhanced algorithm. The second part consists of a damper and a spring element in series. They model the rate dependent behavior and also the reaction of the material to changes in strain rate as shown in Fig.9. Their properties (viscosity η and modulus E) can be functions of strain and strain rate, so that any set of rate dependent stress-strain-curves can be modeled.

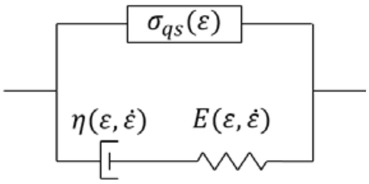

Fig.10: Rheological model used to incorporate viscous effects into the algorithm.

By assuming a constant relaxation time τ of the rheological model (which means that η / E is constant), it can be shown that the algorithm described in section 2 of this paper can be easily modified to include viscous effects. The interpolation equation (1) as well as the interpolation matrix introduced in equation (4) basically stay the same. Only the interpolation factors that are used have to be changed:

$$\sigma_i = (1 - F_i)\tilde{\sigma}_k + F_i \tilde{\sigma}_{k+1} \quad (9)$$

with

$$F_i(t) = \frac{1}{\tau}\int_0^t f_i(s) e^{-\frac{t-s}{\tau}} ds + f_i(0) e^{-\frac{t}{\tau}} \quad (10)$$

with f_i according to equation (2). In this modified algorithm the new interpolation factors F_i are explicitly time dependent. It is also possible to consider more than one relaxation time by using Prony series.

5 Summary

A method was presented which allows for the calculation of input curves for rate dependent material models at arbitrary constant strain rates out of experimental data. The proposed algorithm, which does not involve any curve fitting, has been implemented with a graphical user interface. The results of simulations using those converted input curves show a much better agreement with the experimental stress-strain- and force-displacement-curves.

Furthermore, an enhanced version of the algorithm was introduced which includes the viscous response of polymers. For future work it is planned to perform tensile tests not only at different haul-off speeds, but also at different constant strain rates in order to validate this enhanced algorithm with experimental data.

6 Literature

[1] "LS-DYNA Keyword User's Manual", Volume II, LS-DYNA R7.1, 2014.

[2] Zhang, C. and Moore, I.D.: "Nonlinear Mechanical Response of High Density Polyethylene. Part 1: Experimental Investigation and Model Evaluation", Polymer Engineering and Science, Vol. 37 (2), 1997, 404-413.