

Macroscopic modelling of polymeric materials

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In the last decades, the use of polymeric materials in the automotive industry has increased dramatically. This demand is linked to the good ratio between cost, density and mechanical properties for this class of materials. Moreover, injection modelling allows a very large range of shapes for the polymeric parts used in a car body. This combination of advantages makes polymers an ideal choice for automotive applications such as interior body parts as well as exterior bumpers.

In the recent years, the automotive industry has paid major attention to the safety of the car passengers and pedestrian. To fulfil the targets requested by the standards, numerical simulations are extensively used to obtain cost effective solutions to these requirements. While metallic materials are today rather well understood by the scientific community and properly handled in commercial finite element software, models for polymers are still a source of discussion. Indeed, a short literature survey reveals that several approaches can be employed to describe the relationship between stresses and strains within a polymeric material. Here the focus is placed on an elasto-plastic description of polymers.

Unlike metallic materials, polymers exhibit a rather large strain to yielding (in the order of 1%). This key feature is in conflict with the usual assumption of small elastic strains in constitutive models for metals. It is therefore always questionable to use kinematics issued from hypo-elasticity [1] to describe the constitutive equations required in an accurate material model for polymers. The other alternative to derive the required kinematics is to employ a hyper-elastic formulation [2], which handles the large elastic strains properly.

This work is concerned with the effect of hypo- and hyper-elasticity on the response of several polymeric materials subjected to low and high strain rates loadings. To enable a fair comparison between the two descriptions of the kinematics, the two models solve the same elasto-visco-plastic equations. To account for the pressure sensitivity usually present in polymers, Raghava's yield function is employed:

$$f = \frac{(\alpha - 1)I_1 + \sqrt{(\alpha - 1)^2 I_1^2 + 12\alpha J_2}}{2\alpha} - (\sigma_0 + R) \quad (1.1)$$

where J_2 represents the second invariant of the deviatoric stress tensor, I_1 is the first invariant of the stress tensor, α is a material parameter describing the ratio between the yield stress in compression and tension, and $\sigma_0 + R$ represents the initial yield stress in tension and the work hardening.

The plastic flow rule is obtained using a non-associated approach. The same mathematical formulation is used as for the yield function, see Eq. (1), but α is replaced by another material parameter β . In this work, it is assumed that the work-hardening present in polymers is purely isotropic. A three-terms Voce hardening rule is employed

$$R = \sum_{i=1}^3 Q_i \left(1 - \exp\left(-\frac{\theta_i}{Q_i} p\right) \right) \quad (1.2)$$

where θ_i and Q_i represent respectively the initial hardening modulus and the saturation stress of the Voce terms, and p is the equivalent plastic strain. By appropriate choices of the material parameters θ_i and Q_i , the three-terms relation in Eq. (2) facilitates representation of hardening as well as softening. Visco-plasticity is also a dominant feature of polymers and is included in the following form:

$$\dot{p} = \dot{p}_0 \left\{ \exp\left[\frac{1}{C} \left(\frac{\varphi}{\sigma_0 + R} - 1 \right) \right] - 1 \right\} \quad (1.3)$$

where \dot{p} represents the equivalent plastic strain rate, \dot{p}_0 and C are material parameters and φ is the Raghava equivalent stress.

To extend the range of application of this model a simple ductile damage model is applied. This model assumes that damage is linked to the volume variation implied by the pressure-dependent Raghava plastic flow rule, and is computed in the rate form as:

$$\dot{D} = (1 - D)\dot{\varepsilon}_v^p \quad (1.4)$$

where \dot{D} is the damage rate, D the damage and $\dot{\varepsilon}_v^p$ is the volumetric plastic strain rate.

The set of non-linear equations imposed by the viscoplastic model is solved using a semi-implicit backward Euler algorithm. This method was chosen for its simple implementation within a hyper-elastic framework. The same approach is applied to the hypo-elastic case.

The parameters of the proposed material model are easily identified based on tensile and compression tests. This simplifies its potential use in an industrial context because any reverse engineering method is not required. The model is implemented into LS-DYNA R6.1.2 through a user-defined material model subroutine (*USER_MATERIAL_MODEL). All simulations were carried out using a double precision smp solver.

To evaluate the differences between the two descriptions of the kinematics, a quasi-static three-point bending test carried out on a 3 mm thick PVC plate is simulated. Figure 1a shows the response in terms of force displacement while Figure 1b shows the distribution of equivalent plastic strain at a displacement of 10 mm. The two models exhibit similar responses, even though the force level is slightly higher for the hyper-elastic material model. A somewhat lower plastic strain field is obtained with the hyper-elastic model compared to the hypo-elastic one.

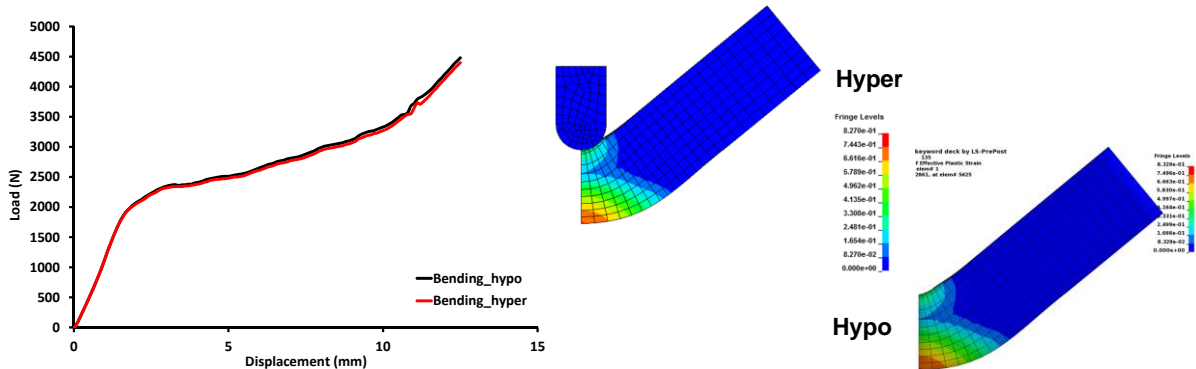


Fig. 1: Quasi-static 3 points bending tests of a PVC plate, a) force-displacement and b) equivalent plastic strain responses.

For this particular example, the hypo-elastic material model is approximately 30% faster than the hyper-elastic formulation. This difference is believed to be link to the more complex mathematical operations carried out within a hyper-elastic framework.

Additional validation cases on a mineral filled polypropylene (PP) and a high density polyethylene (HDPE) have shown the same tendencies as in the presented case on PVC. For low and moderate levels of plastic strain (i.e., lower than 1), the two approaches give similar answers while an increase in computational time is present for the hyper-elastic approach. One conclusion that can be drawn is that for filled polymers (PP for instance) exhibiting reasonable ductility the use of a hyper-elastic constitutive model is not required while very ductile polymers such as HDPE would benefit of such approach. Within an industrial context, the use of hypo-elastic models is normally an optimal choice when computational time is taken into account.

Literature

- [1] Balieu R., Lauro F., Bennani B., Delille R., Matsumoto T., Mottola E., "A fully coupled elastoviscoplastic damage model at finite strains for mineral filled semi-crystalline polymer", International Journal of Plasticity, 51, 2015, 241-270
- [2] Polanco-Loria M., Clausen A.H., Berstad T., Hopperstad O.S., "Constitutive model for thermoplastics with structural applications", International Journal of Impact Engineering, 37, 2010, 1207-1219