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State of the Art in Simulation of Composite Structures

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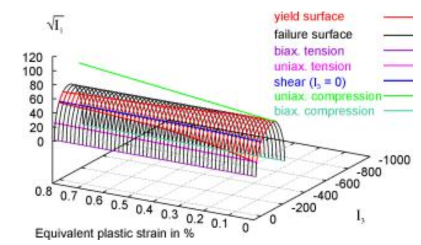
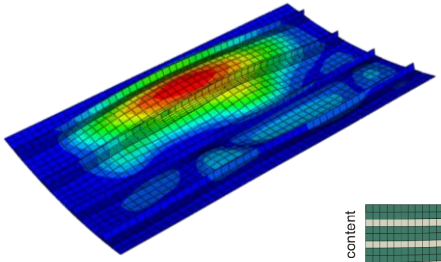
Introduction

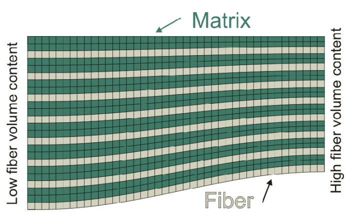
- Increasing application of composites (aerospace, wind energy, automotive).
- Considerable progress in the last two decades has been made in simulation capability for composite structures, but the level has by far not yet reached the level for isotropic structures.
- The success of composites, in particular for advanced applications, depends on the availability of reliable, accurate, and economically efficient prediction methods.

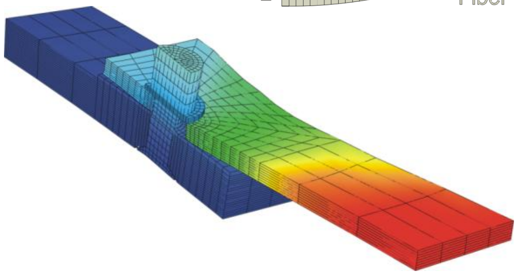

Challenges

- What are the challenges?
  - Inhomogenity and anisotropy (fiber, matrix, nanoparticle)
  - Complex failure behavior (fiber failure, matrix failure, delamination, interface failure, progressive failure)
  - Various imperfections (geometric imperfections, fiber waviness, porosity)
  - Joining methods currently used are not the most suitable for composite material, and increase the complexity of the analysis.



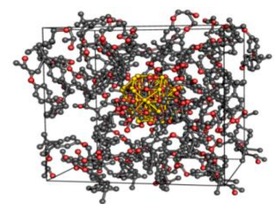


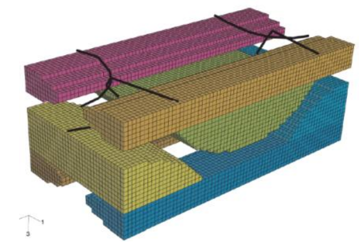


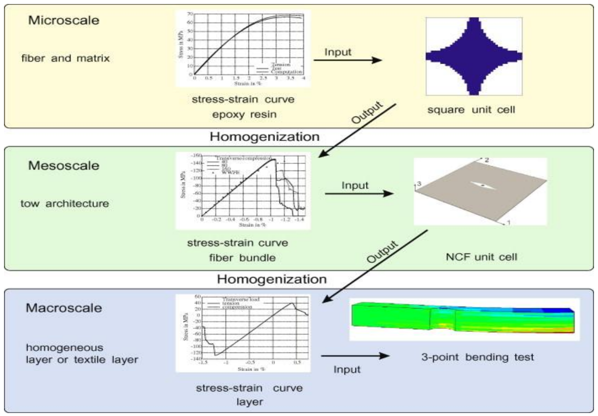


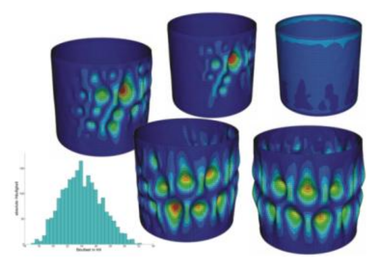
Solutions?

- What are the solutions?
  - Gain a better understanding of composite materials (a direct transfer from isotropic material to composite material is not possible).
  - Look “deeper” into the material (both analytically/numerically and experimentally).
  - Progressive failure analysis.
  - Efficient probabilistic methods.
  - Joining methods suited for composite materials.









Contents

- **Constitutive Modeling:** *Modeling Pressure Dependent and Rate Dependent Pre-Failure Nonlinearities*
- **Strength:** *Simulating the Effect of Porosities on Stiffness and Strength*
- **Stability:** *Semi-Analytical and Numerical Probabilistic Buckling Analysis of Composite Shells*
- **Fatigue Analysis:** *A Physics-based Fatigue Approach for Composites Combining Failure Mechanisms, Strength and Stiffness Degradation*

Novel Transversely Isotropic Elastic-Viscoplastic Constitutive Law

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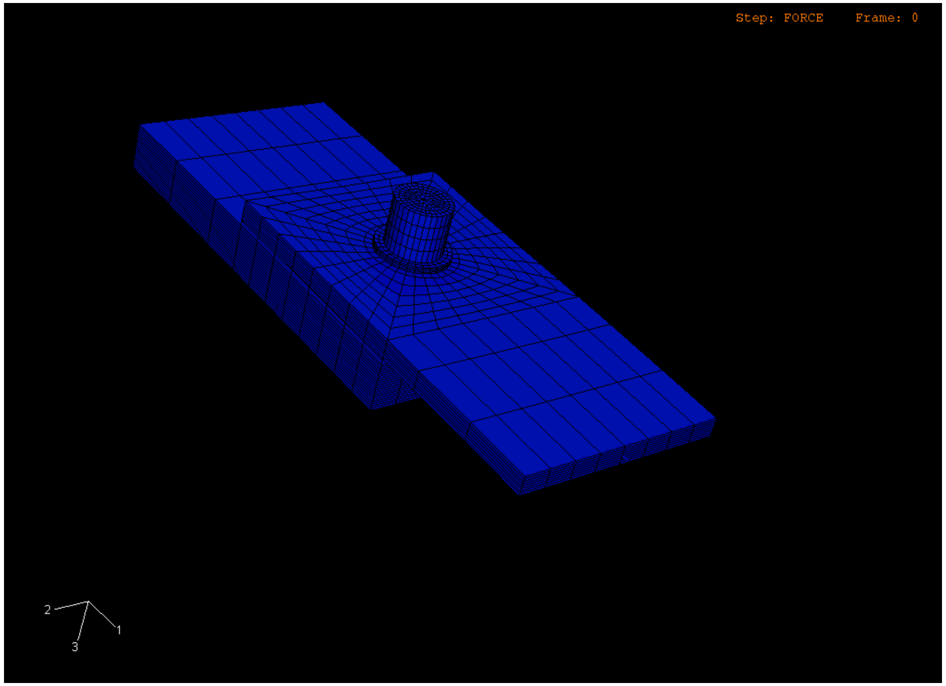
Application: UD fiber matrix composites



<u>Objective:</u> Simulation of all pre-failure nonlinearities in all loading states

•Plasticity based nonlinearities in combined compression-shear stress states

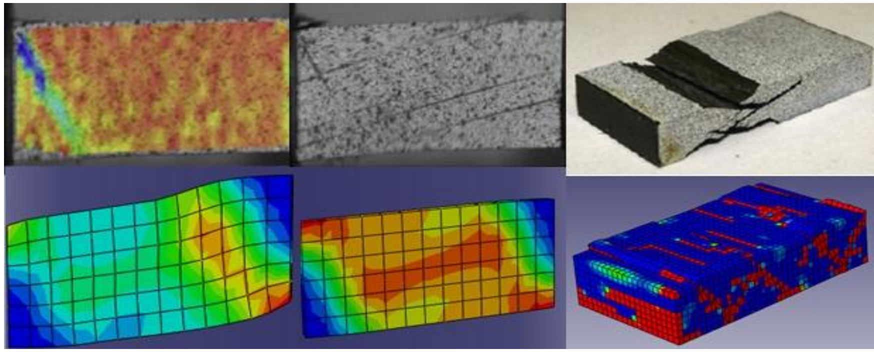
•Example: Boltes Joints

Quasi-plastic deformations at hole edge cause redistribution of loads in a row of bolted joints

Novel Transversely Isotropic Elastic-Viscoplastic Constitutive Law

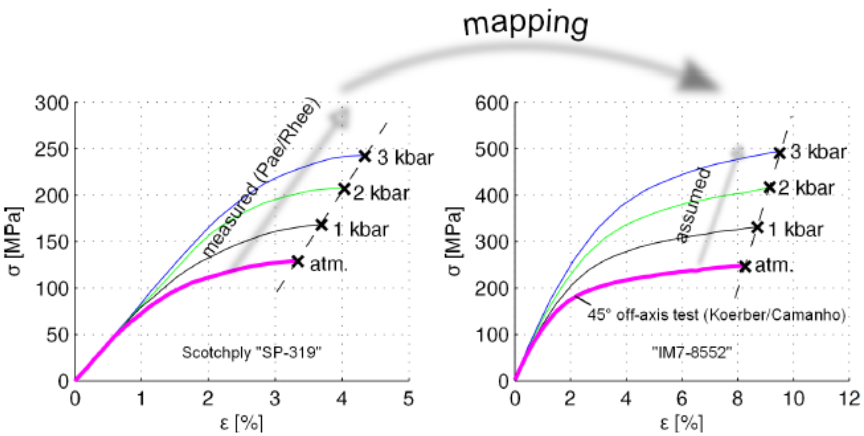
UD carbon-epoxy: **IM7-8552**

•Quasi-static and dynamic off-axis compression tests



Tests: Camanho/Körber

•Uniaxial compression tests under various levels of hydrostatic pressure



Tests: Pae/Rhee

Transversely Isotropic Elastic-Viscoplastic Constitutive Law

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- **Yield surface**
  transversely isotropic invariants used

$$f = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 - 1$$

- **Visco-plastic formulation**
  (Cowper-Symonds)

$$\sigma_y \;\; \dot{\varepsilon}, \varepsilon_p \;\; = \sigma_y \;\; 0, \varepsilon_p \left[1 + \left(\frac{\dot{\varepsilon}}{C} \right)^{\frac{1}{p}} \right]$$

$$\alpha_3 = \alpha_3^t \quad , \alpha_{32} = \alpha_{32}^t \quad \text{if} \quad I_3 > 0$$

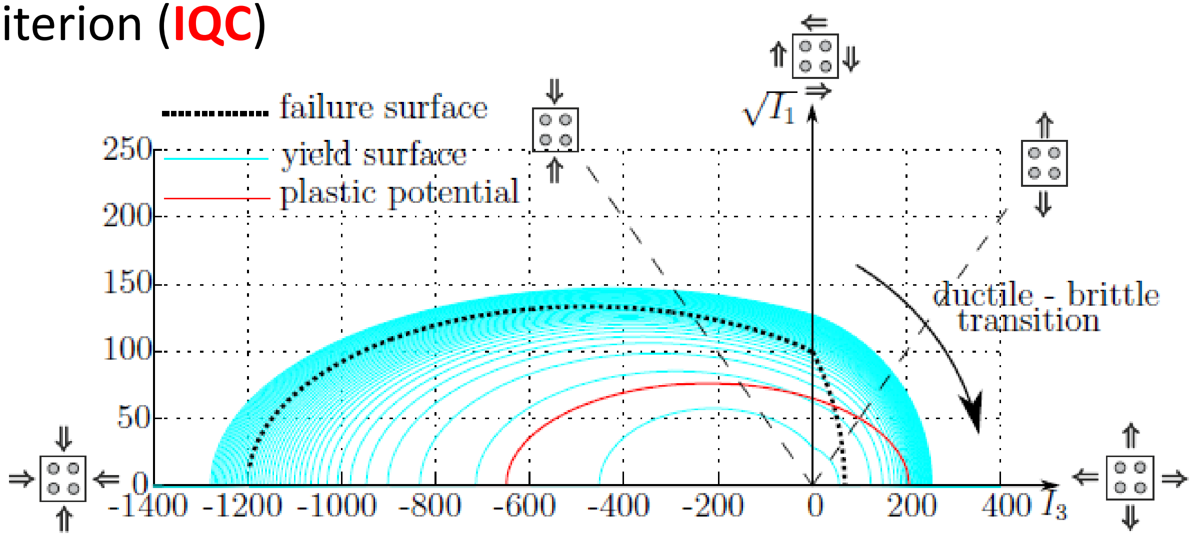
$$\alpha_3 = \alpha_3^c \quad , \alpha_{32} = \alpha_{32}^c \quad \text{if} \quad I_3 \leq 0$$

$$F = \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_{32} I_3^2 = 1$$

- **Interfiber failure**
  Invariant-based **Q**uadratic **C**riterion (**IQC**)
  (in analogy to yield surface)

- **Fiber failure**

$$F = \frac{\mathbf{a}\boldsymbol{\sigma}\mathbf{a}}{R_{\|}^{t}} = 1 \qquad F = \frac{\mathbf{a}\boldsymbol{\sigma}\mathbf{a}}{-R_{\|}^{c}} = 1$$



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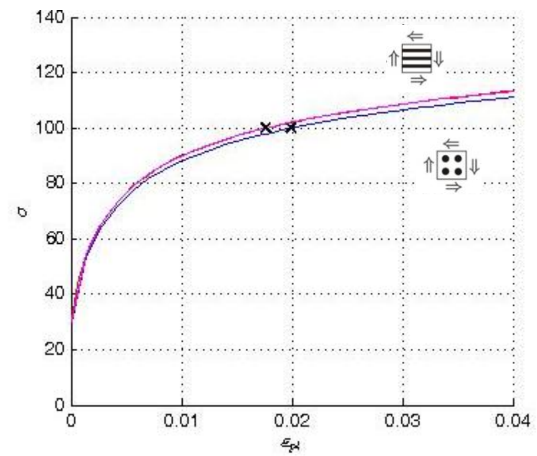
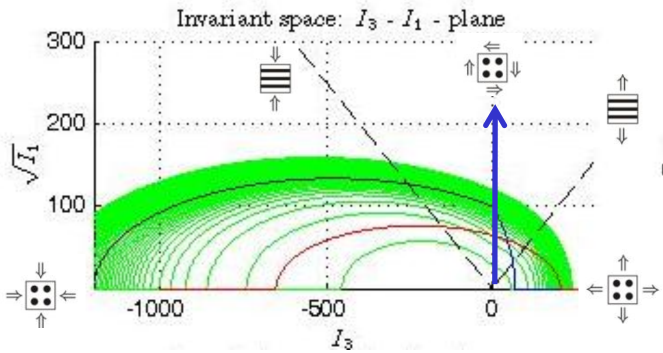
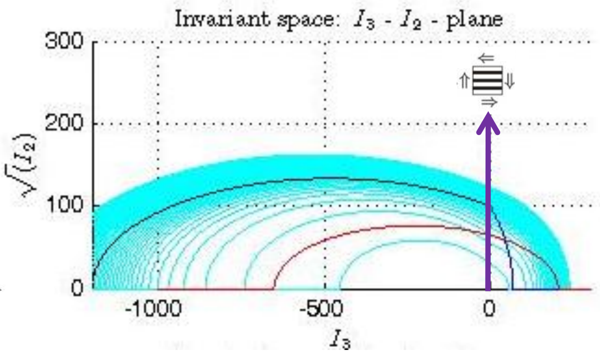
Yield and Failure Surface for IM7-8552
- Invariant Representation -

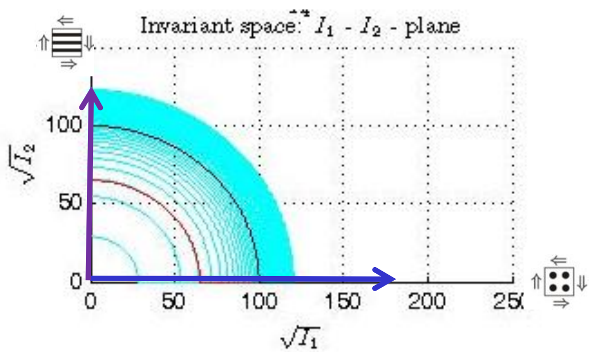
Yield surface $$f = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 - 1$$

Failure surface $$F = \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_{32} I_3^2 = 1$$





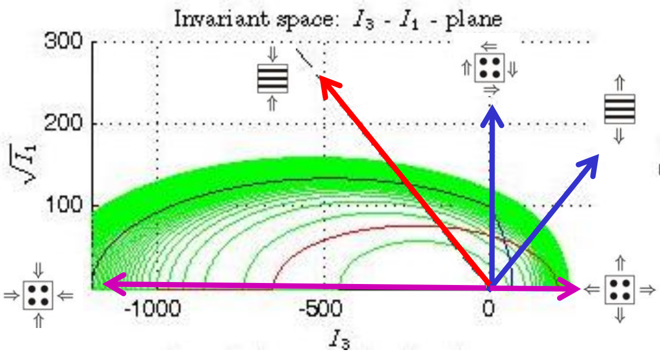


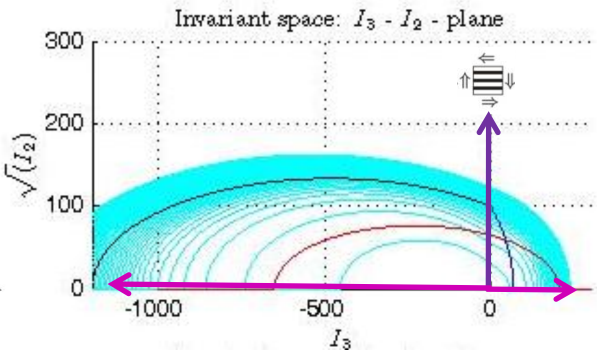


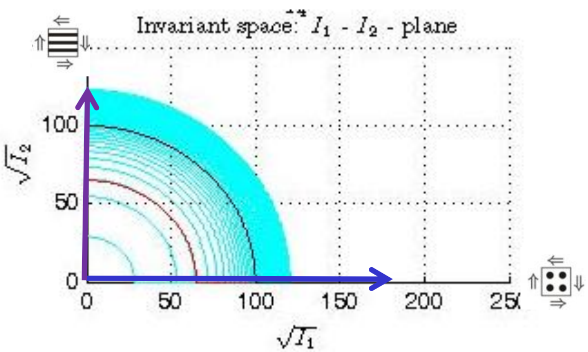
Yield and Failure Surface for IM7-8552
- Invariant Representation -

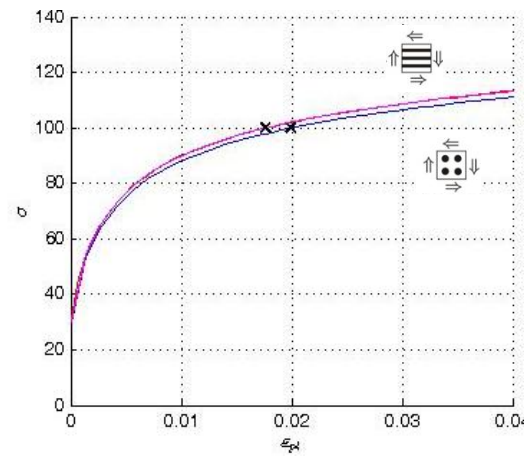
Yield surface
$$f = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 - 1$$

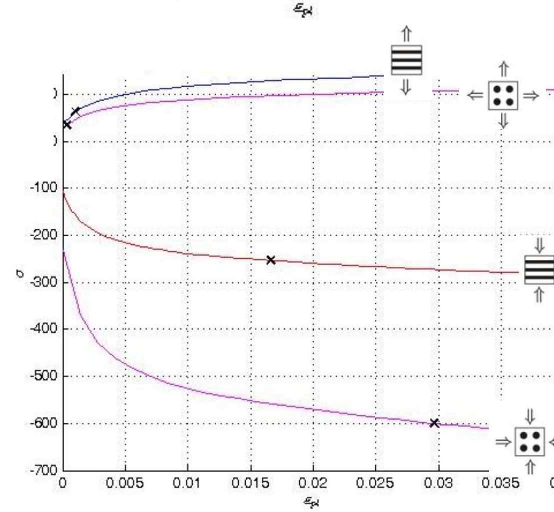
Failure surface
$$F = \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_{32} I_3^2 = 1$$











Off-Axis Tests IM7-8552
- Quasi-Static and Dynamic -

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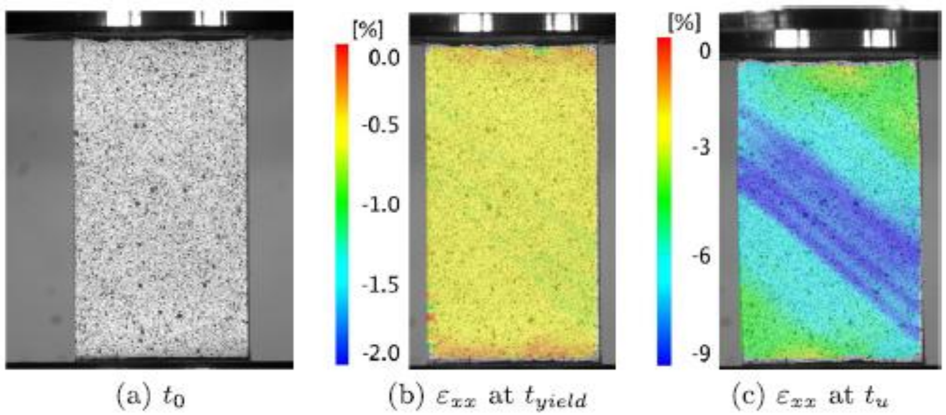
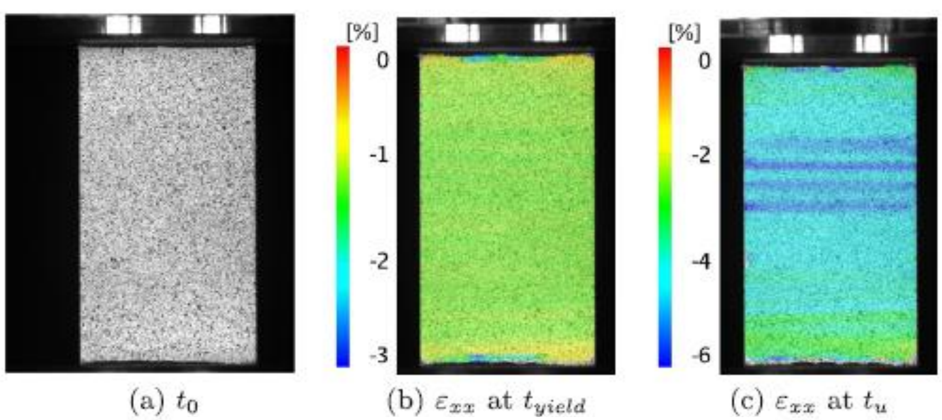
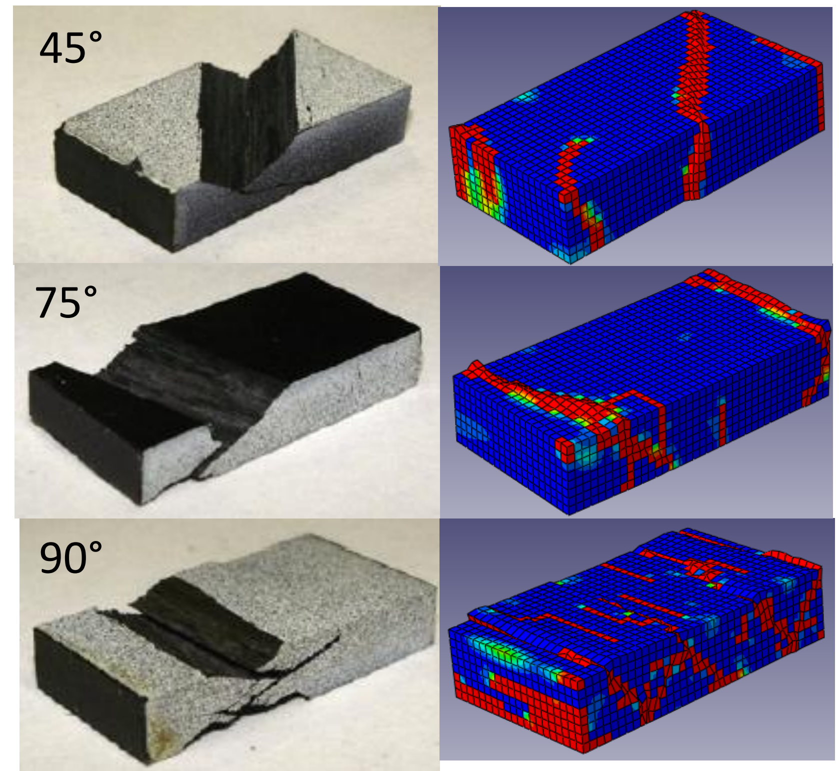
15° / 30° / 45° / 60° / 75° / 90°



Quasi-static 45° off-axis test



Quasi-static 90° off-axis test



Tests: Hannes Körber / Pedro Camanho.

Off-Axis Tests IM7-8552
- Simulation Results -

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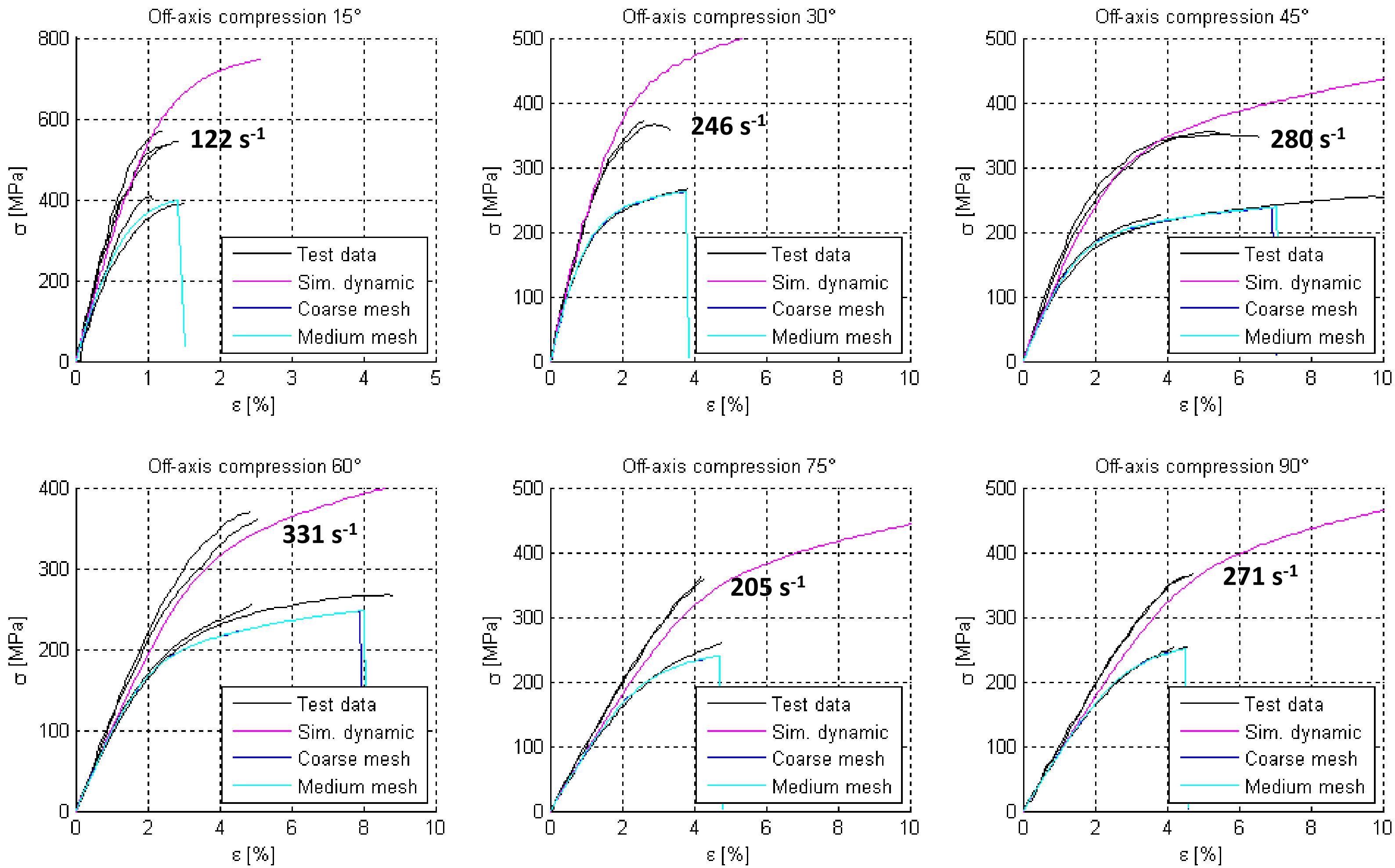
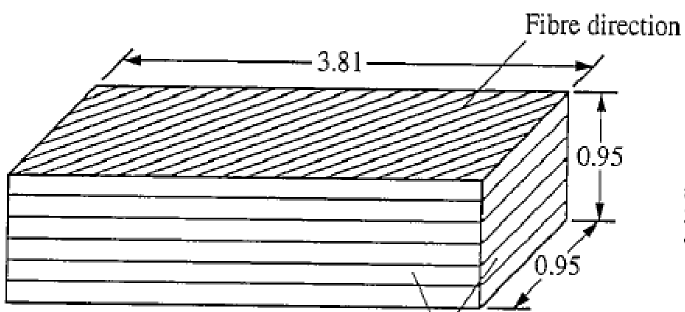
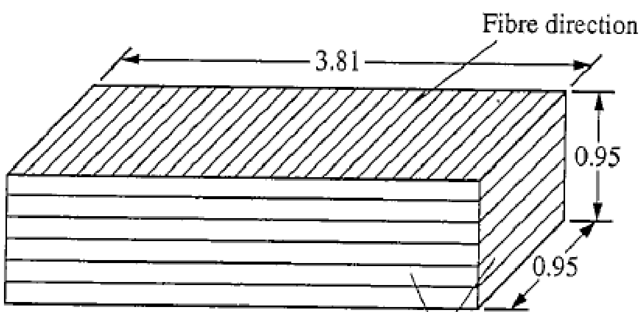


High Pressure Tests acc. Pae/Rhee

Test specimen

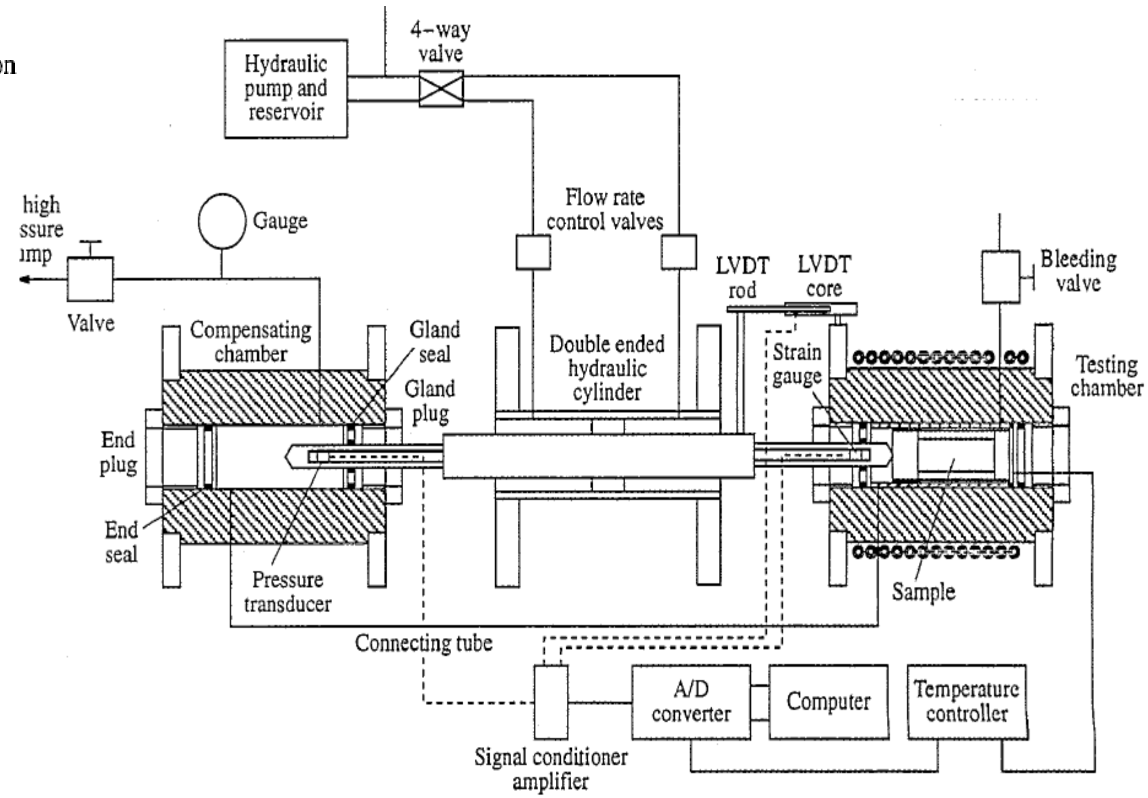


45° sample



90° sample

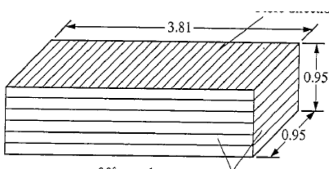
Test apparatus



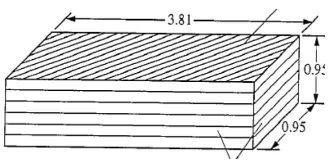
K.D. Pae & K.Y. Rhee : „Effects of hydrostatic pressure on the compressive behavior of thick laminated 45° and 90° unidirectional graphite-fiber/epoxy matrix composites“

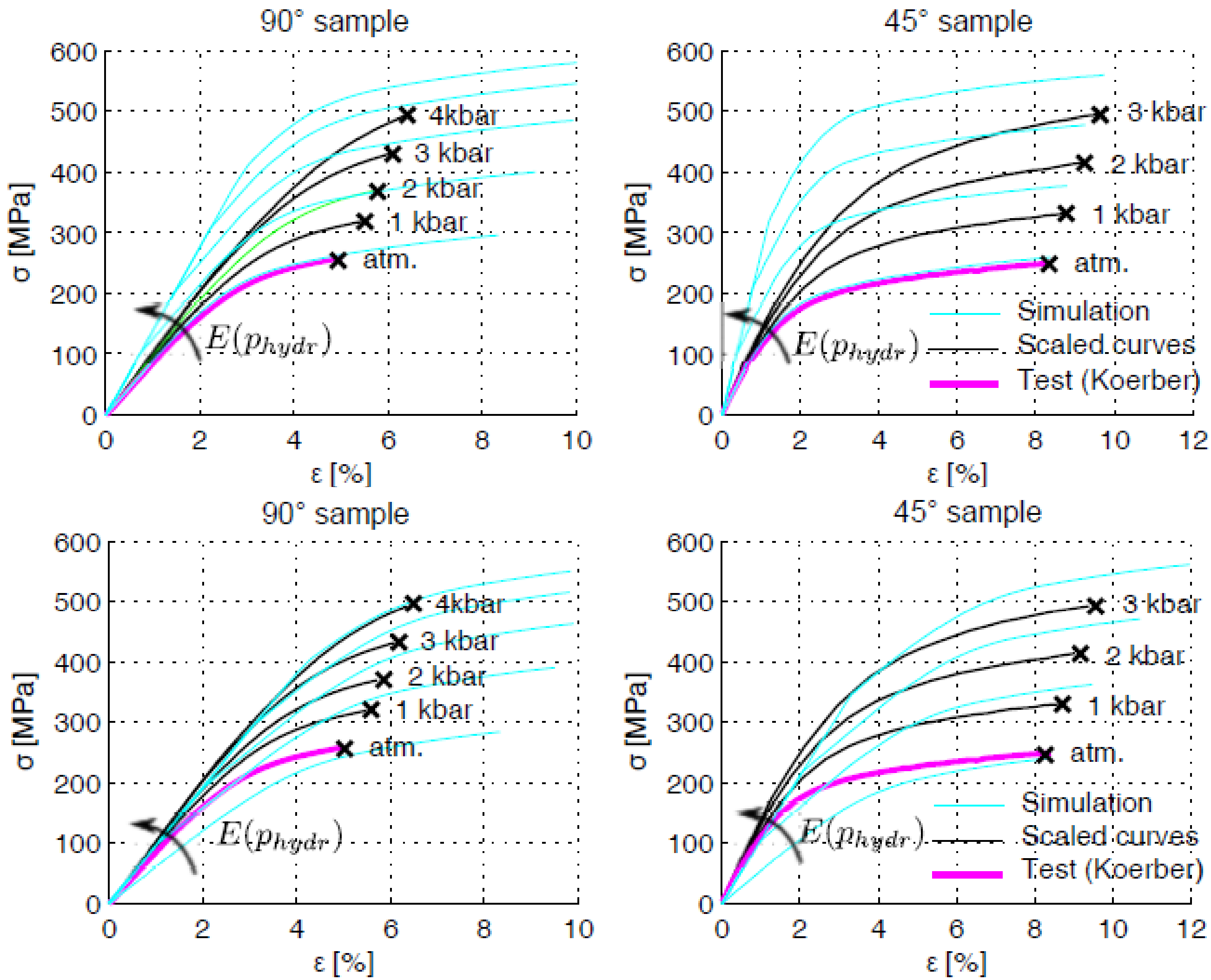
Carbon Epoxy Composites under High Hydrostatic Pressures

90° sample

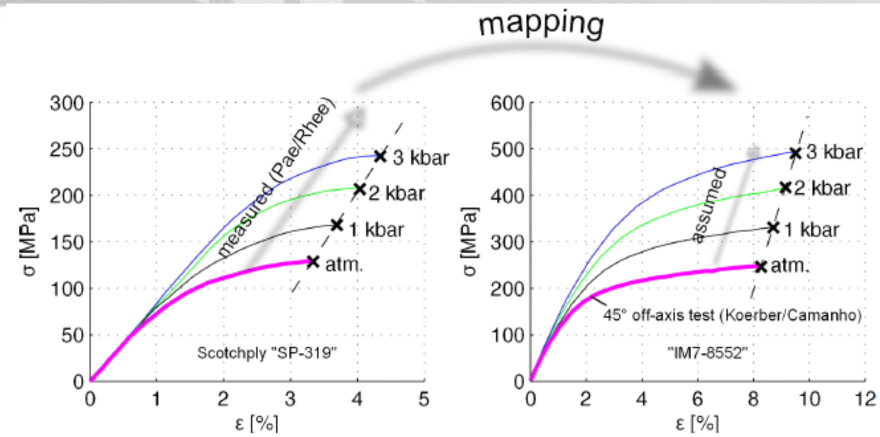


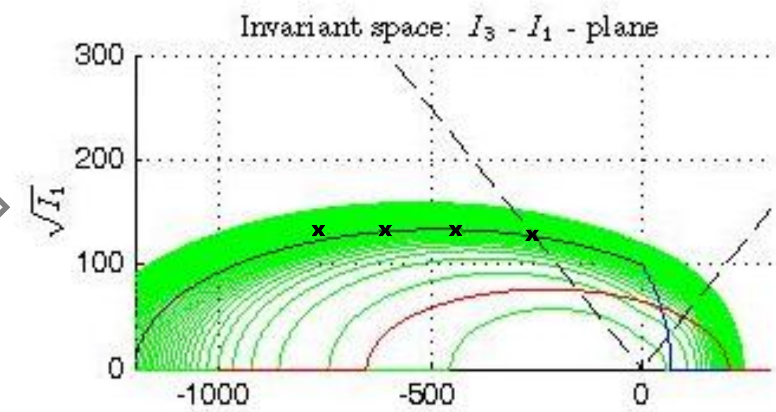
45° sample

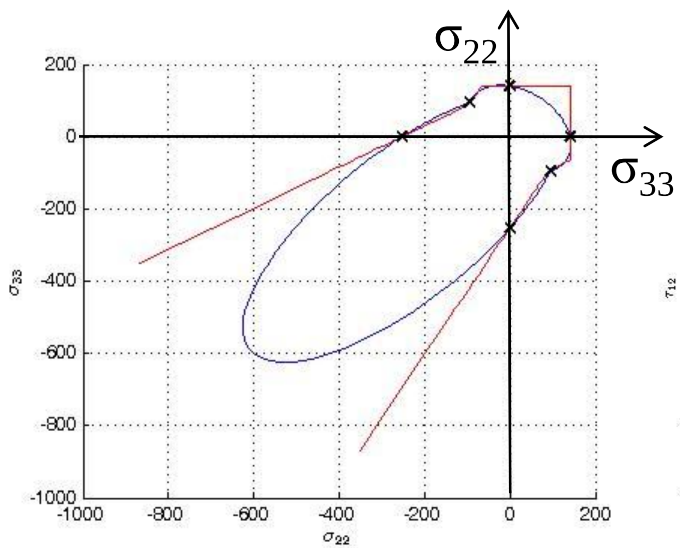


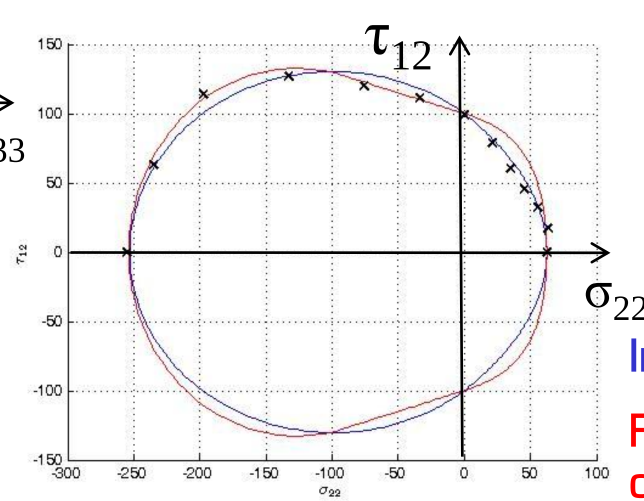


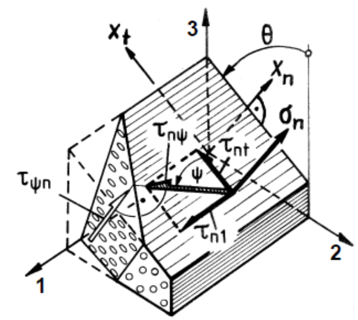
Failure Criteria











Invariant based criterion

Fracture Criterion, based on stress vector in fracture plane

Novel transversely isotropic constitutive model

- Prefailure nonlinearities can be regarded
- Behavior of composites under high hydrostatic pressures is approximated
- Strain rate dependent behavior captured by visco-plastic approach (Cowper-Symponds model)

Current work:

- Addressing strain rate effects in failure, softening and plasticity:
  ➡ Cooperation with group from Pedro Camanho, Universidade do Porto
  - Rate dependent failure surfaces
  - Rate dependent fracture toughness

  } Experiments in progress
- Coupling of transversely isotropic viscoplastic law (Vogler/Rolfes) with smeared crack model (Camanho et al.)

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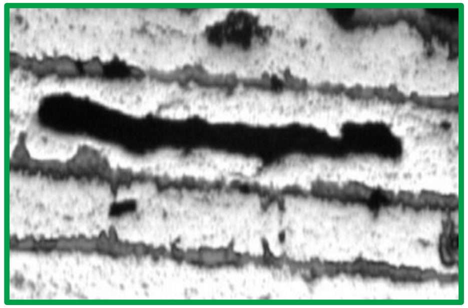
Motivation

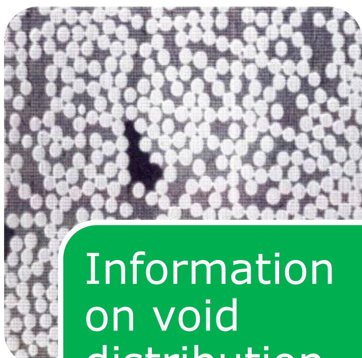
- Production defects can not be avoided (without dramatically increasing the production costs)
- Voids have detrimental effect on
  - Stiffness
  - Strength



- Prediction of material properties of imperfect laminates is the basis for economic design
- Void content is measured by ultrasonic attenuation
  → no information on void morphology
- Analytical methods exist to predict elastic properties but are also generaly based on void content only

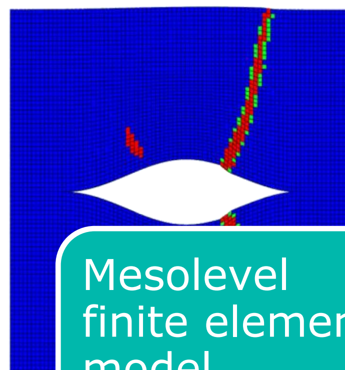
Analysis Concept

- Objective: replacing experimentally obtained knock-down values by accurate numerical predictions



Information on void distribution (TUHH)

- size
- shape
- Location



Mesolevel finite element model

- Nonlinear material model
- Progressive failure

Parameters for macroscale

- Stiffness
- strength

Multiscale simulation

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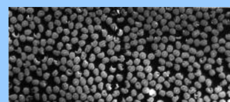
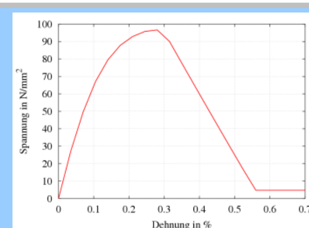
Micro

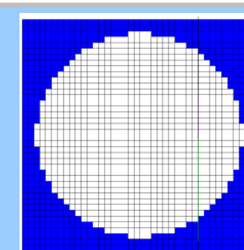
Fiber and matrix





Material behavior fiber and matrix

Input



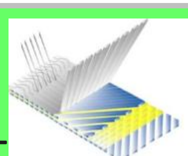
Unit cell fiber bundle

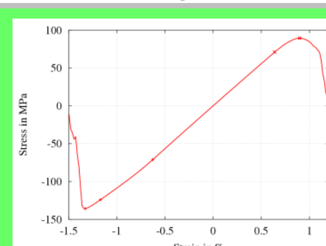
Output

Homogenization

Meso

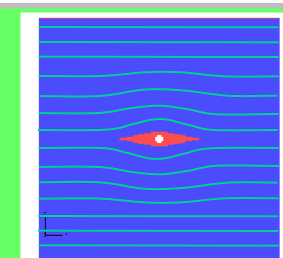
Fiber architecture





Material behavior fiber bundle and matrix

Input



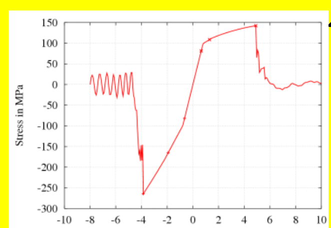
Unit cell non-crimp fabric

Output

Homogenization

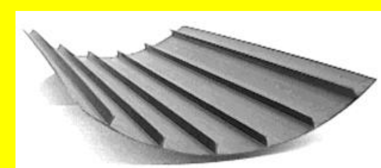
Macro

Homogeneous layer



Material behavior of one layer

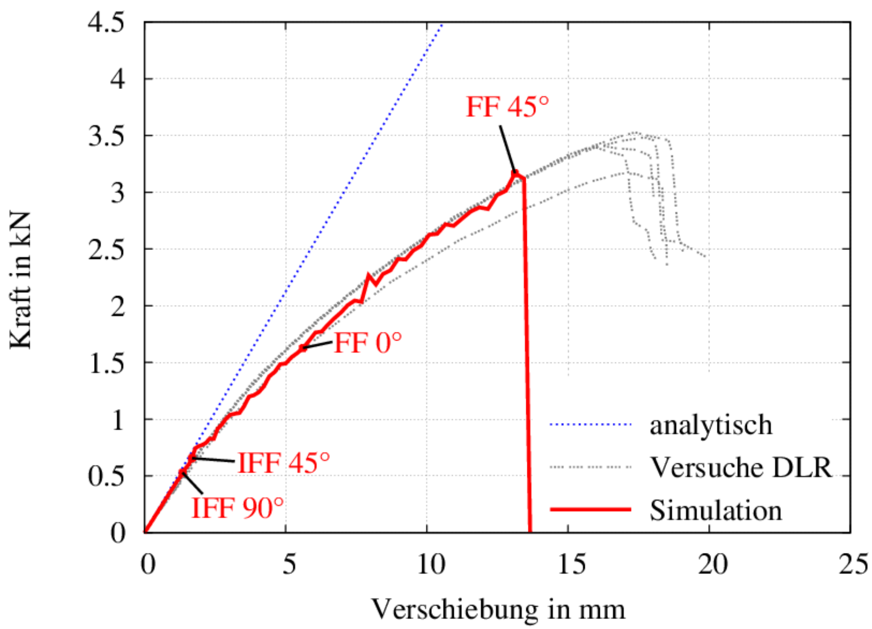
Input

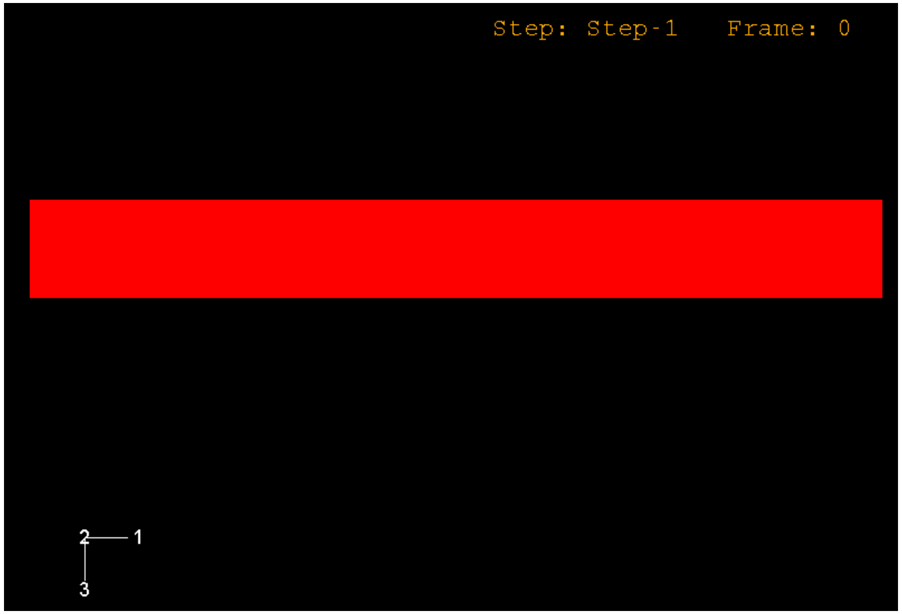


[Source: DLR]

Three-Point Bending Test

- Characteristic damage state
- Progressive damage
- Failure by combination of fiber failure in 45 -plies and delamination

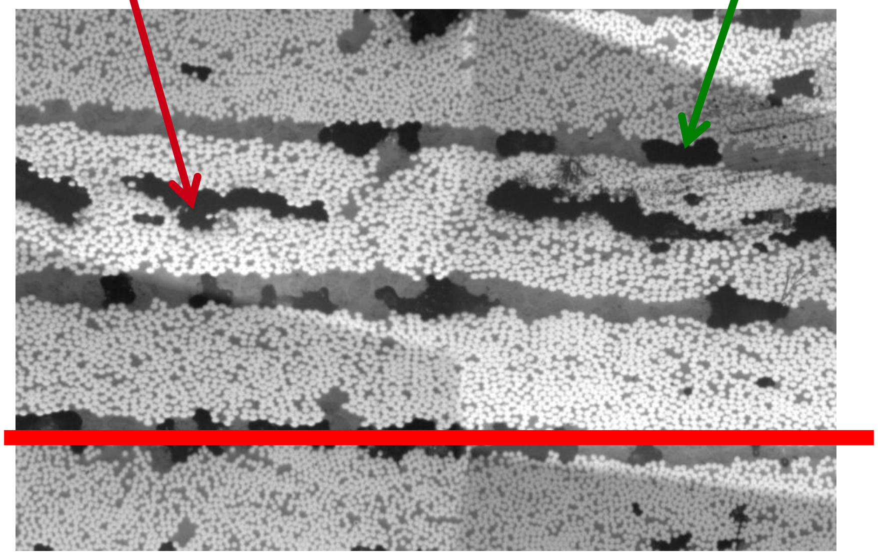


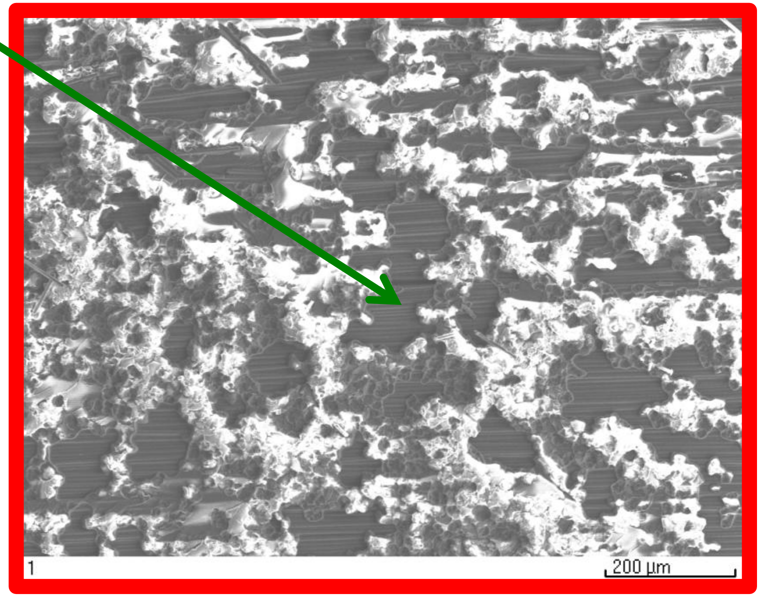


Void Classification

Voids inside layer

Voids between layers





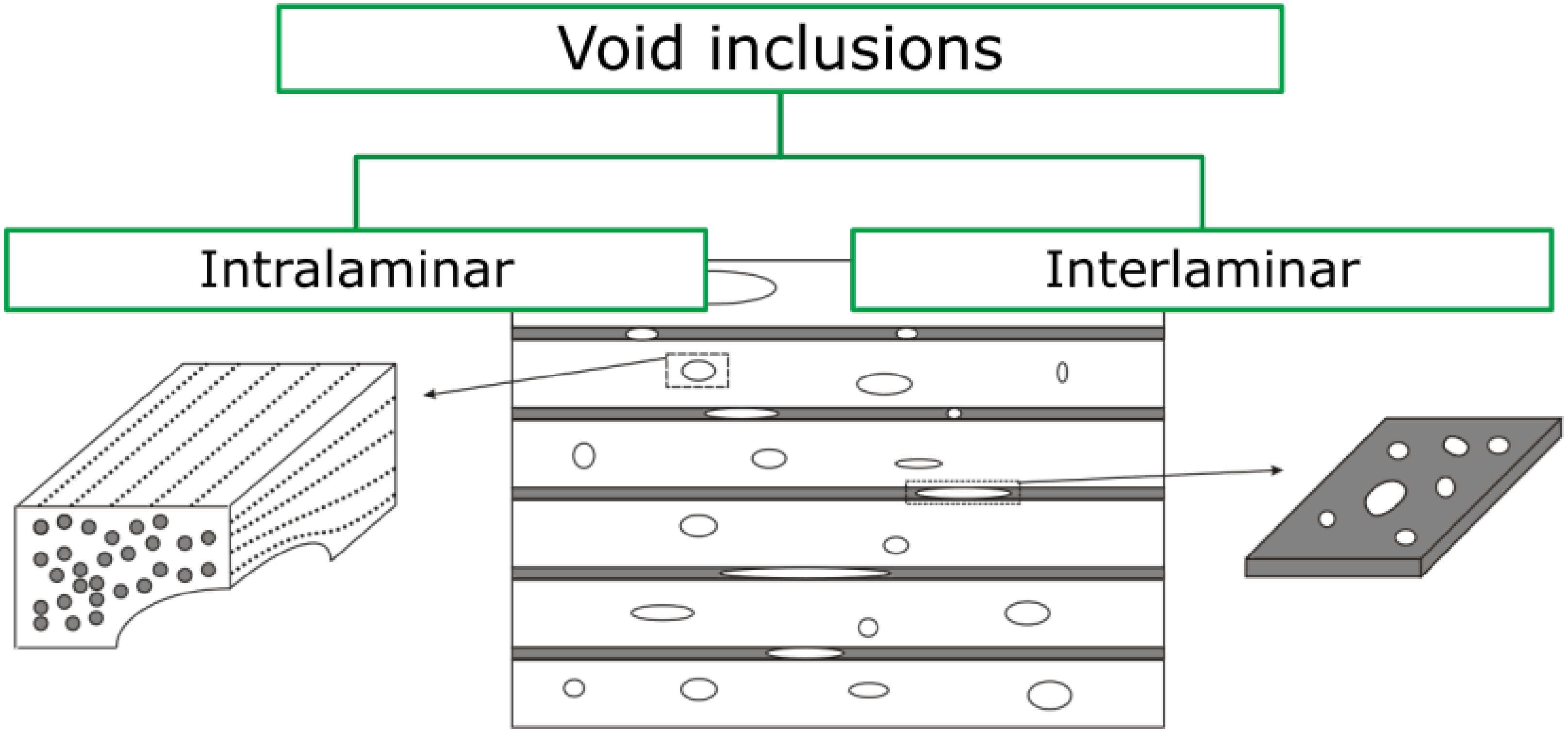
cross section of laminate

z
y

porous resin layer

y
x

Void Classification

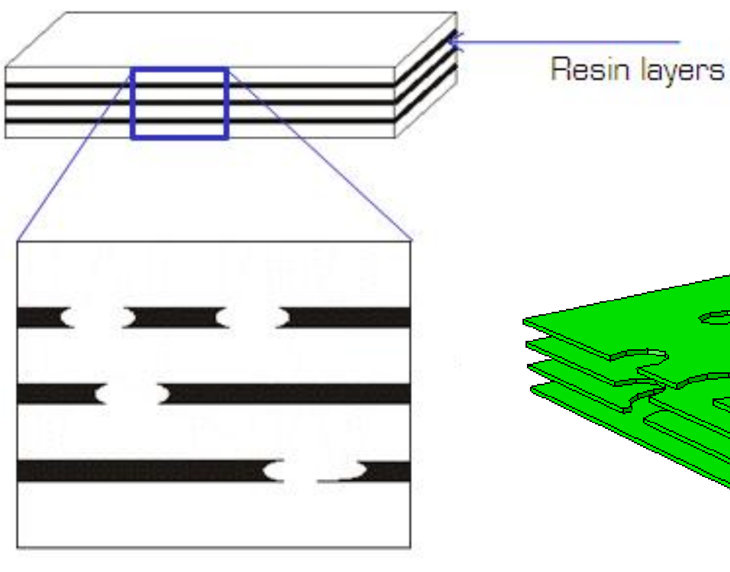


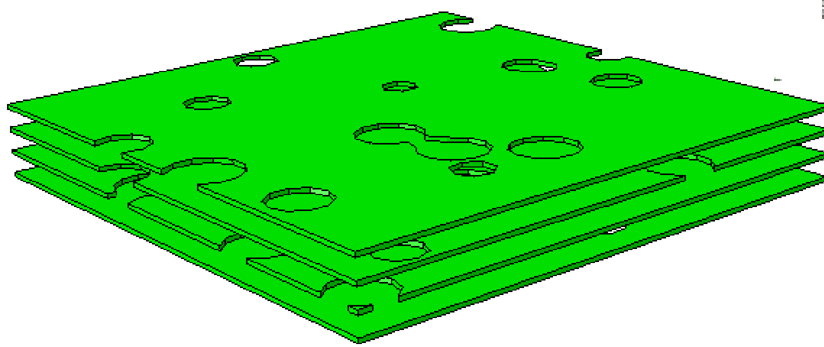
- Oriented in fiber direction
- Voids cause fiber undulations
- Elliptical or cigar-like shape

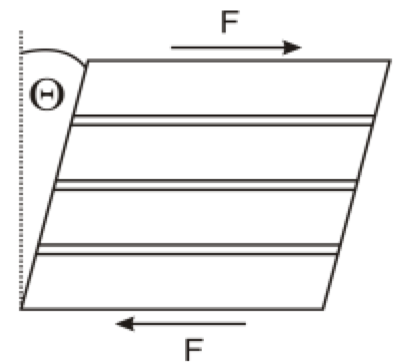
- No preferred orientation
- Independent from ud-layers
- Arbitrary shapes

Finite element model for interlaminar voids

- Four layers under shear loading
- Continuum elements to model the resin layer
- Voids are created at randomly selected position, with randomly selected size
- Voids are allowed to overlap → more general shapes
- Void content and average size of void are varied

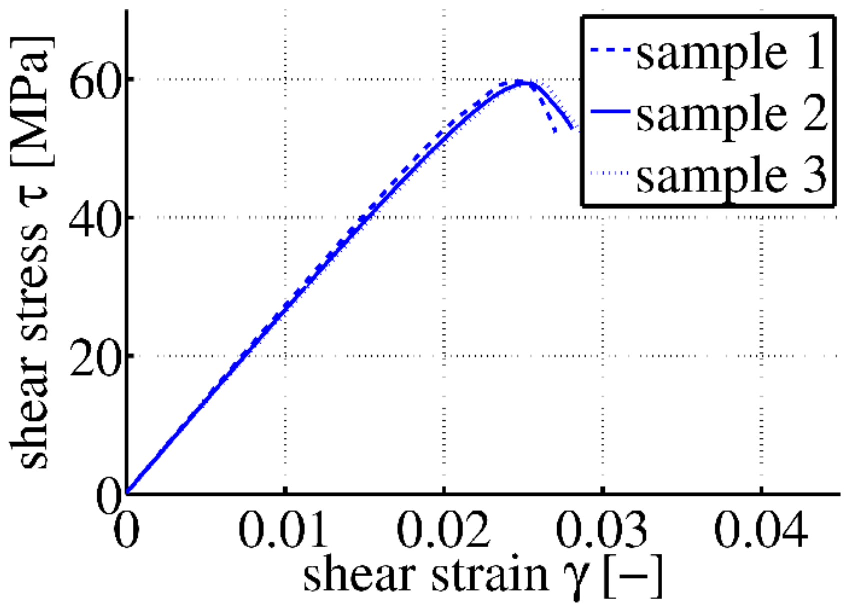


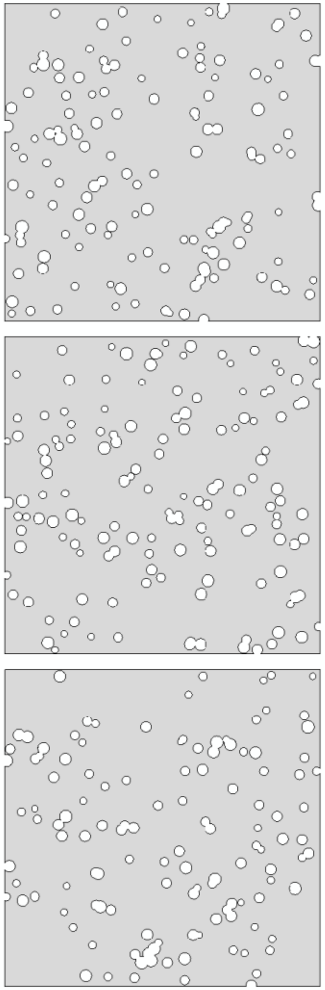




Effect of random distribution

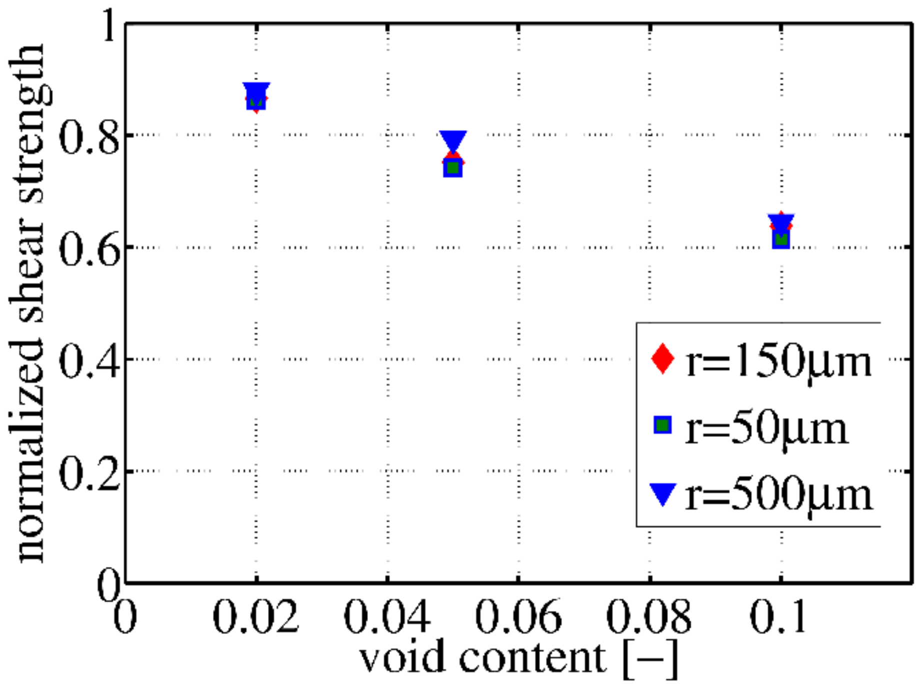
- Different realizations of same void content (10%) and average void size (150 µm)
- Macroscopic stress-strain relation does not differ significantly





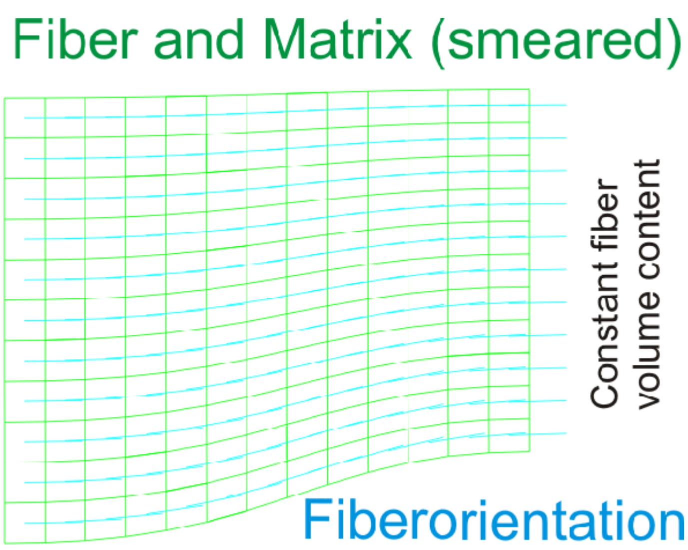
Effect of void content

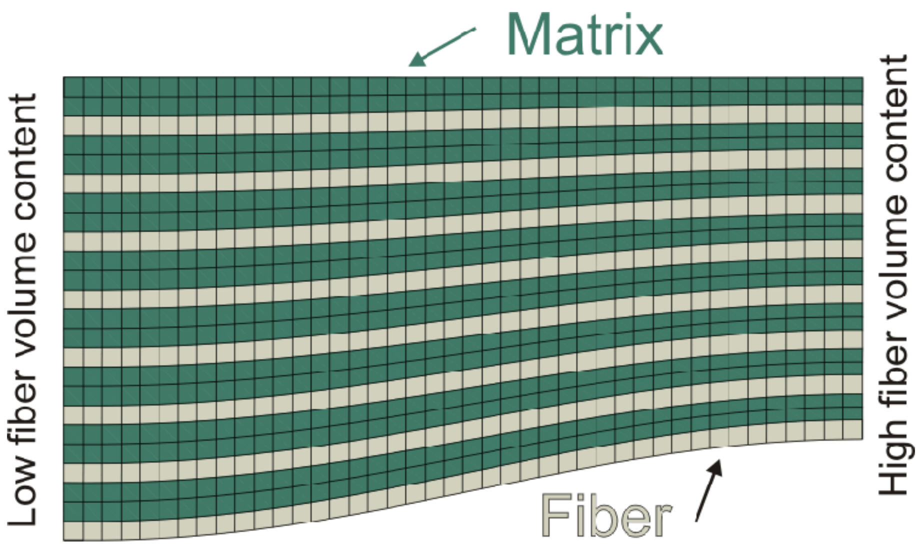
- Significant influence of void content on shear strength
- Small influence of average void size
- Uniform distribution of void radii in the interval [0.75*x, 1.25*x] around mean radius x



Finite Element Model for Intralaminar Voids

- Void inclusions cause fiber undulations
- Compression load case is considered
- Two levels of refinement are used:
  - Smeared modeling of fibers and matrix
  - Discretization of single fibers





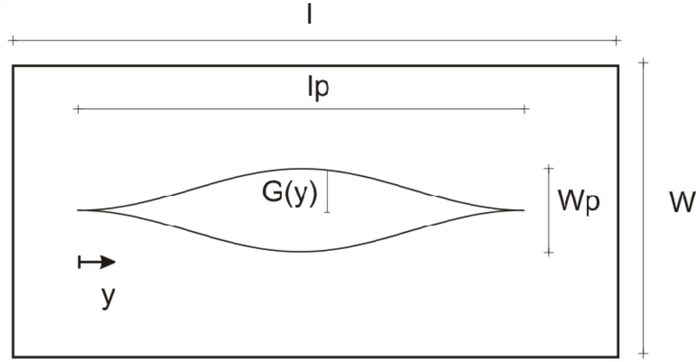
Results: Intralaminar Voids

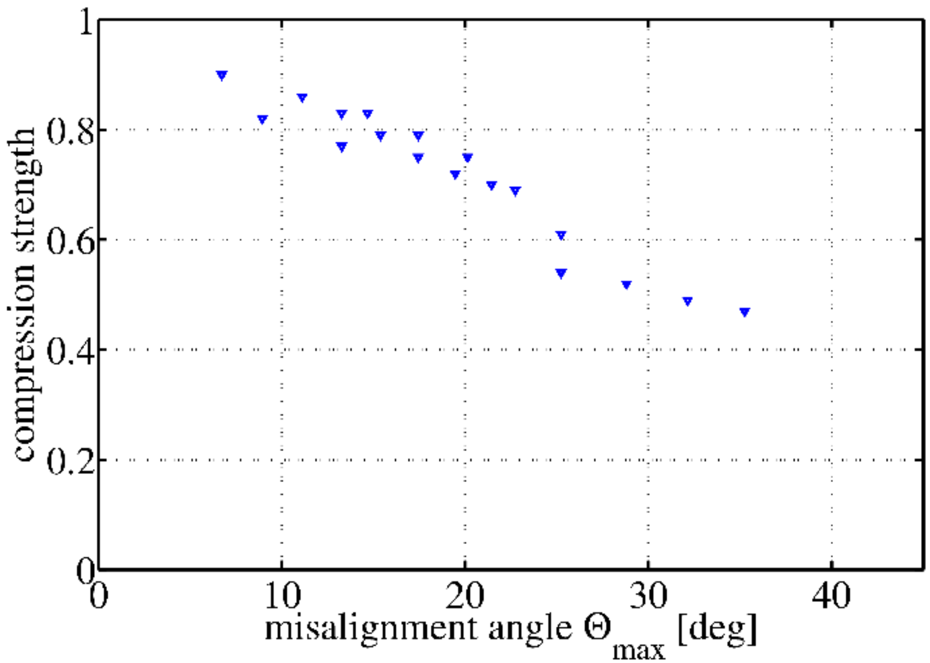
- Variation of width and length
- Fiber misalignment angle dominates compression strength

$$g = \pm\frac{w_p}{2}\left(1 - \cos\left(\pi\frac{y}{l_p}\right)\right)$$

$$\Theta_{max} = \arctan\left(\frac{w_p\pi}{2l_p}\right)$$

width [mm]	length [mm] 1.00	1.50	2.00
0.15	0.73	0.82	0.90
0.2	0.70	0.83	0.86
0.3	0.54	0.79	0.83
0.35	0.52	0.75	0.79
0.4	0.49	0.69	0.75
0.45	0.47	0.61	0.72





Conclusions

- A numerical model for the prediction of strength reductions due to void inclusions has been presented
- Void content of interlaminar voids dominates shear failure
- fiber misalignment angle causes drop in compression strength

Outlook

- Evaluation of void geometry from micrographs
- Creation of a full 3D model
- Experimental validation

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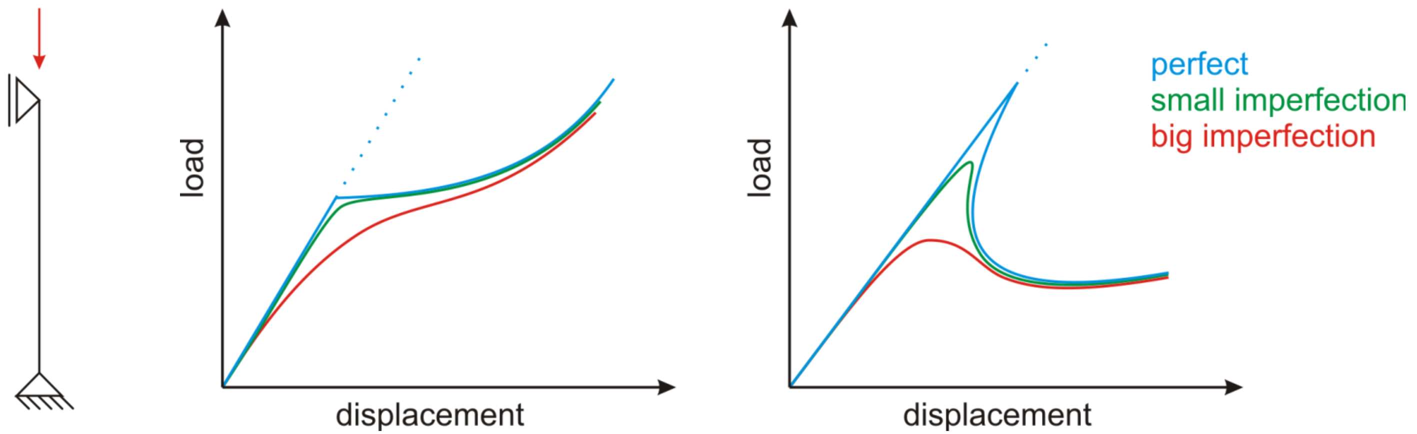
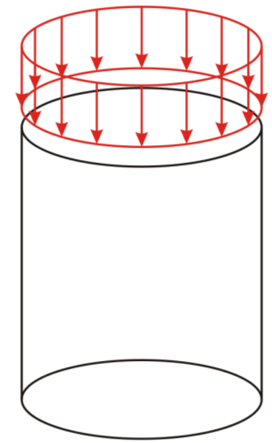
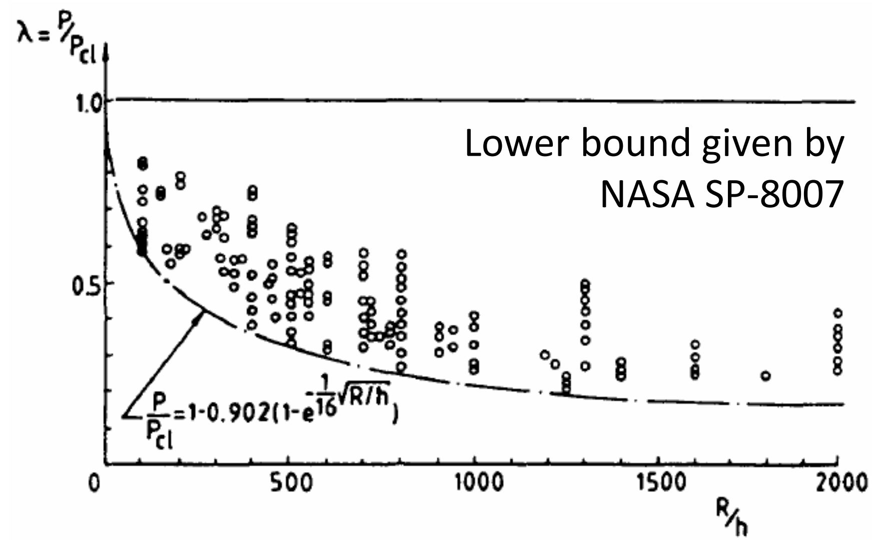
Introduction





- Buckling loads of circular shells depend on imperfections
- Existing guidelines turned out to be very conservative
- Not intended for composite shells (No consideration of laminate setup)



Lower bound given by NASA SP-8007

Imperfections

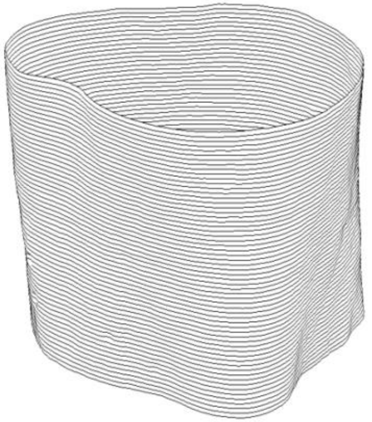
Considered set of 10 CFRP cylinders:

- Manufactured, measured and tested at DLR in Braunschweig
- Optical measurement of geometric imperfections
- Ultrasonic measurement of wall thickness
- Coupon tests for material characterization

Characteristics of shells considered

Laminate setup: [24 , 41]
Radius: 250 mm
Length: 500 mm
Thickness: 0.5 mm
Radius/Thickness: 500

Imperfections



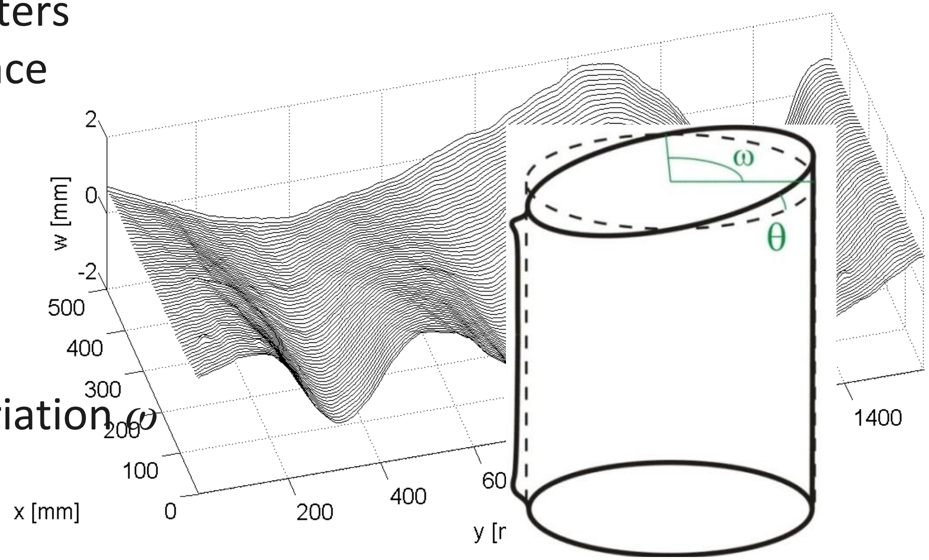
Imperfections considered

- Geometric imperfections, described by Fourier series

$$\bar{W}(x,y) = t\sum_{k=0}^{n_1}\sum_{l=0}^{n_2}\xi_{kl}\cos\frac{k\pi x}{L}\cos\left(\frac{l\,y}{R}-\varphi_{kl}\right)$$

→ $n_1 \cdot n_2 \cdot 2 = 462$ correlated parameters (ξ_{kl} and φ_{kl}) describe the shell surface

- Material parameters E_{11}, E_{22} and G_{12}
- Wall thickness t
- Bending angle θ and circumferential variation ω



Imperfections

Mahalanobis transformation:

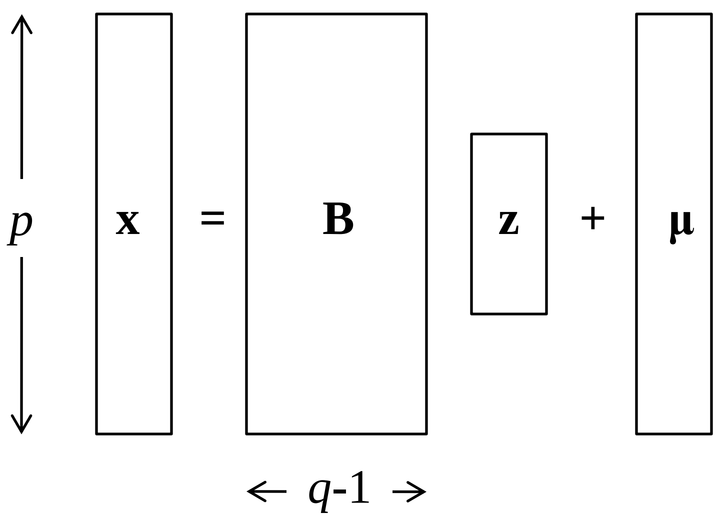
$$\mathbf{x} = \boldsymbol{\Sigma}^{\frac{1}{2}}\,\mathbf{z} + \boldsymbol{\mu} \quad \text{and} \quad \mathbf{z} = \boldsymbol{\Sigma}^{-\frac{1}{2}}\;\mathbf{x} - \boldsymbol{\mu}$$

One dimensional equivalent:

$$x = \sigma\, z + \mu \quad \text{and} \quad z = \frac{x - \mu}{\sigma}$$

If

number of random parameters p (here: p = 462) > number of measurements q (here q = 10)



The root $\mathbf{B}$ is otained from spectral decomposition of $\boldsymbol{\Sigma}$

$$\mathbf{B} = \mathbf{U}\,\mathbf{D}^{\frac{1}{2}} = \boldsymbol{\Sigma}^{\frac{1}{2}}$$

$\mathbf{X}_g$: 462 random parameters
→ $\mathbf{Z}_g$: 9 random parameters

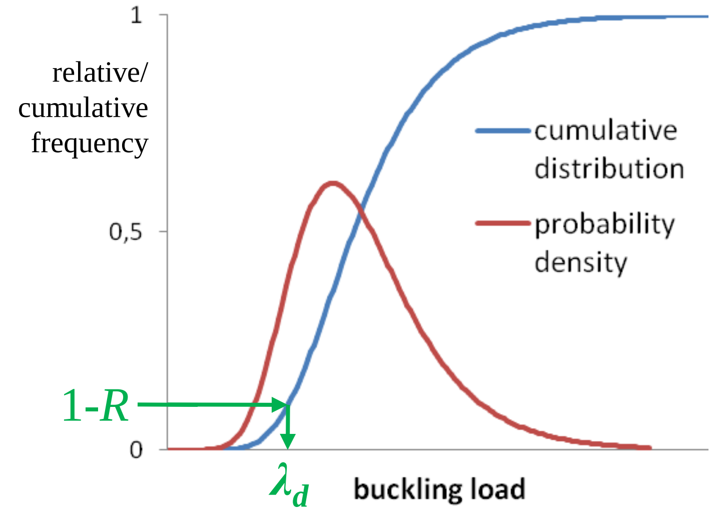
Probabilistic Analysis

Probabilistic design procedure:

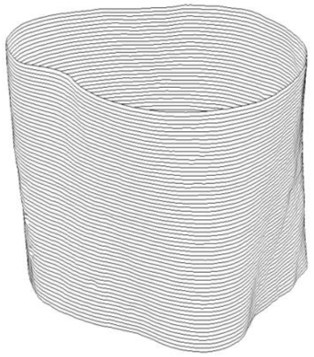
Determine the stochastic distribution of the buckling load

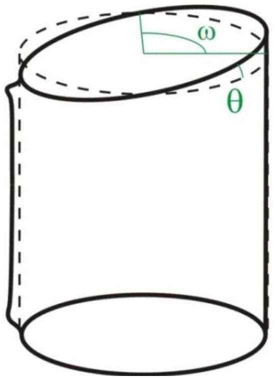
Choose a level of reliability R (e.g. 99 %)

Define the associated buckling load λ_d as design load



- 15 random variables considered
  - 9 independent geometry parameters z_i
  - Material parameters E_{11}, E_{22} and G_{12}
  - Wall thickness t
  - Bending angle θ and circumferential variation ω





Semi-Analytical Probabilistic Analysis

Semi-analytic approach:

- Approximation of buckling load function $\lambda(\mathbf{x})$ by Taylor expansion at mean vector $\boldsymbol{\mu}$ of $\mathbf{X}$

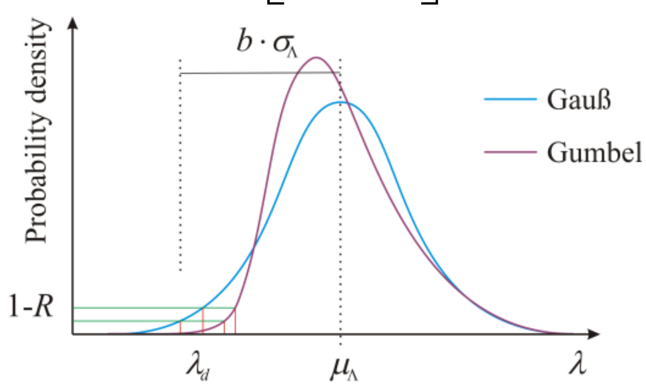
$$\lambda(\mathbf{x}) = \lambda(\boldsymbol{\mu}) + \sum_{i=1}^{n} \frac{\partial \lambda(\boldsymbol{\mu})}{\partial x_i}(x_i - \mu_i) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 \lambda(\boldsymbol{\mu})}{\partial x_i\, \partial x_j}(x_i - \mu_i)(x_j - \mu_j) + \ldots$$

- Determine characteristic moments of the distribution of buckling load

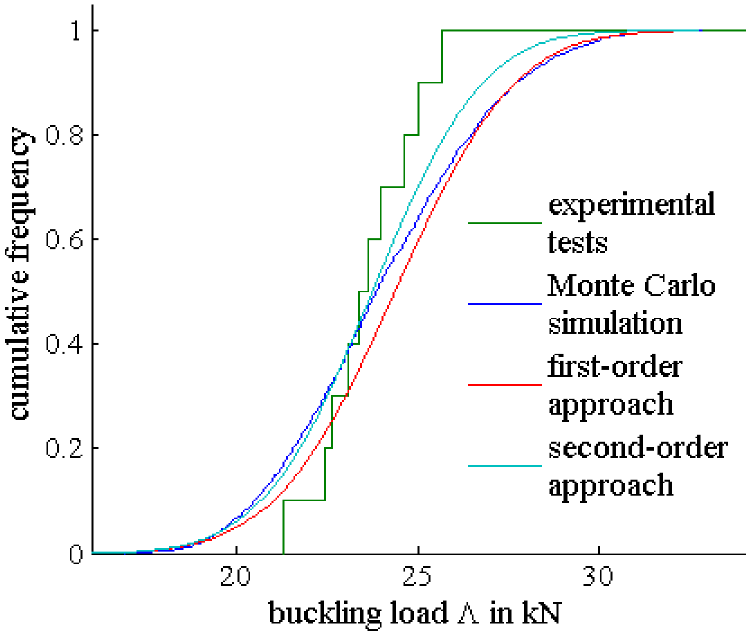
$$\mu_\Lambda \approx \lambda(\boldsymbol{\mu}) + \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 \lambda(\boldsymbol{\mu})}{\partial x_i^2} \operatorname{var}(X_i) \qquad \sigma_\Lambda^2 \approx \sum_{i=1}^{n} \left[\frac{\partial \lambda(\boldsymbol{\mu})}{\partial x_i}\right]^2 \operatorname{var}(X_i)$$

- Choose a type of distribution and level of reliability to obtain the design load

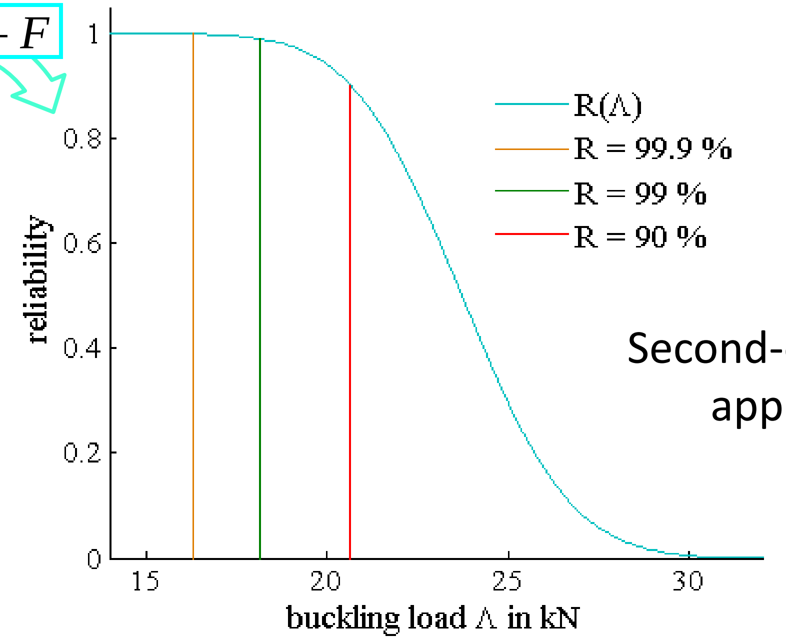
$$\lambda_d = \mu_\Lambda - b \cdot \sigma_\Lambda$$



Semi-Analytical Probabilistic Analysis



$R = 1 - F$



Second-order approach

Reliability	Buckling load in *kN*		
	First-order	Second-order	Monte Carlo
99.9 %	16.3	16.3	17.4
99 %	18.3	18.1	18.6
90 %	21.0	20.7	20.5
NASA SP-8007		10.2	
Min. Test result		21.3	

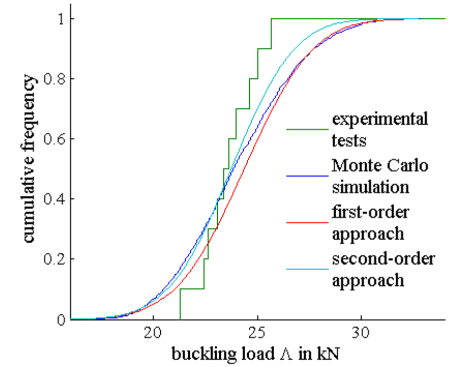
Conclusions

Probabilistic approach	Semi-analytic	Monte Carlo simulation
Assumptions	$f(\Lambda)$	$f_X(\mathbf{X})$
→ evaluated by	$v(\Lambda)$	measurements and K-S test
Number of evaluations of $\Lambda(\mathbf{x})$	$2 \cdot rn + 1 = \boxed{31}$	convergence study → $\sim \boxed{1300}$
Direct Result	$E(\Lambda)$, $var(\Lambda)$, $v(\Lambda)$	$F(\Lambda)$

Probabilistic analyses predict the real distribution well

Semi-analytical methods reach the same accuracy as numerical methods

Probabilistic analysis regarding decisive imperfections leads to an efficient design load



R = 99.9 %	16.3 kN
R = 99 %	18.1 kN
Min. Test result	21.3 kN

Outlook

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Application to stiffened composite panels

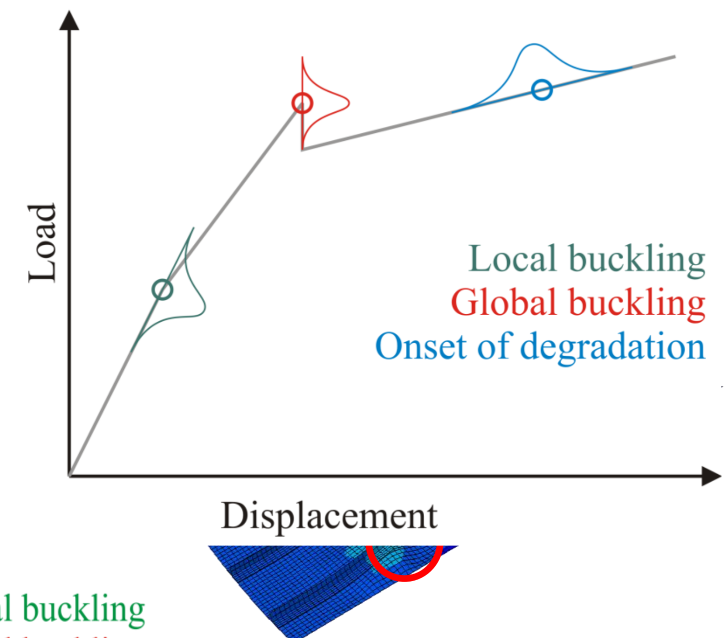
- Complex buckling behavior: interaction of stability failure and material failure
- Design driving: onset of material degradation and global buckling

  → two correlated objective functions

- Enhancement of the proposed procedure for a fast determination of the correlation of global buckling and onset of degradation

$$\rho_{LB-GB} \approx \frac{1}{\sigma_{\Lambda_{LB}} \cdot \sigma_{\Lambda_{GB}}} \sum_{i=1}^{n} \frac{\partial \lambda_{LB}}{\partial x_i}\bigg|_{\boldsymbol{\mu}} \frac{\partial \lambda_{GB}}{\partial x_i}\bigg|_{\boldsymbol{\mu}} \operatorname{var} X_i$$

→ Probability of failure from joint distribution



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Phenomenological Development of Fatigue Damage

Damage =

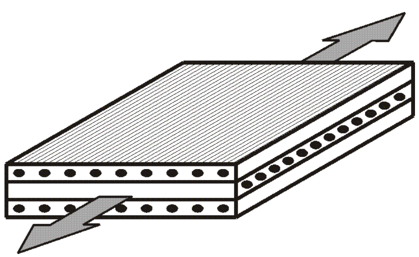
Matrix cracks: $f_I(x)=A_I \cdot \ln(x+B_I)+C_I$ — off-axis-layer

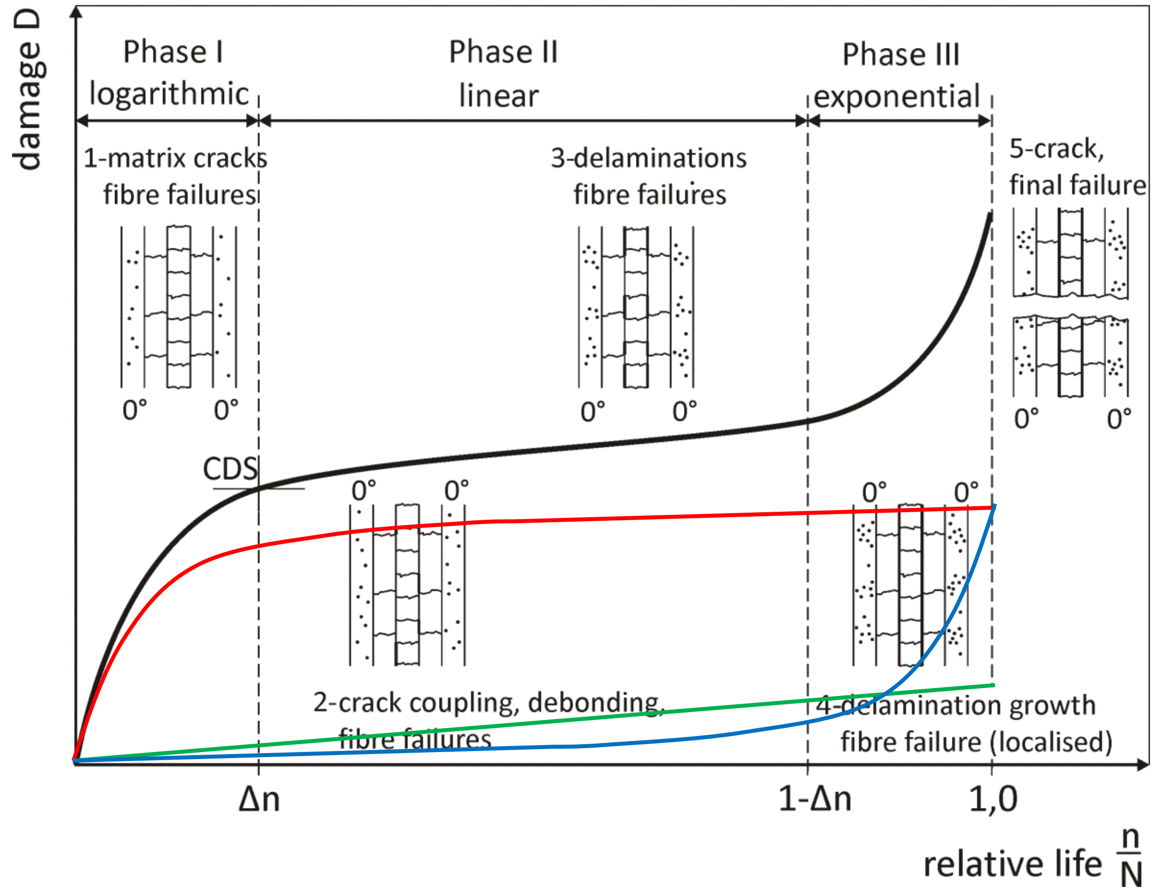
+

Delaminations: $f_{II}(x)=A_{II} \cdot x$ — between layers

+

Fibre failure: $f_{III}(x)=A_{III} \cdot e^{(BIII.x+CIII)}$ — on-axis-layer





Damage-evolution-curve [REIFSNIDER, 1990]

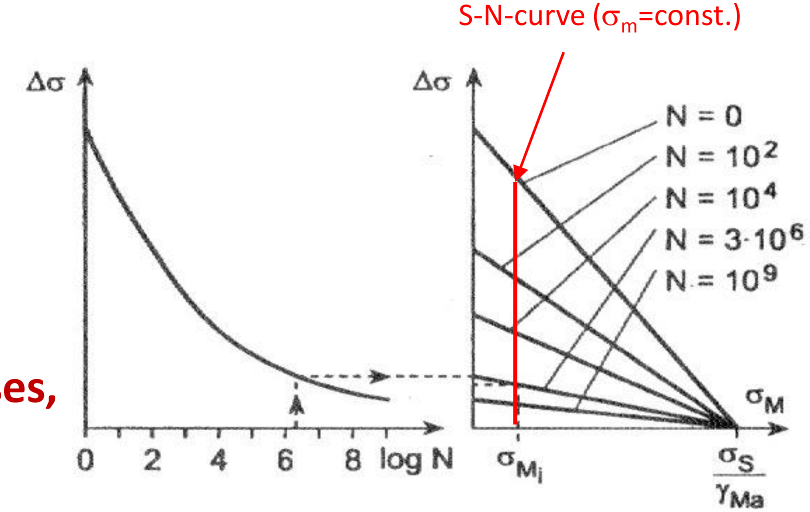
Uncoupled FE-analysis and fatigue analysis:

1. Determination of local stresses in main direction on **laminate level**
2. Application of **empirical** constant-life-diagrams (SN-curve, Goodman or Haigh) and determination of the number of cycles to failure N
3. Linear damage accumulation (Palmgren-Miner):

$$D = \sum_i \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots \leq 1$$

→ **Consequence:**

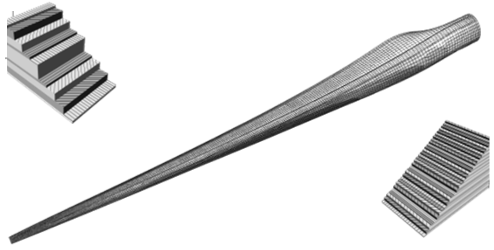
Each laminate configuration (lamina thicknesses, number of layers, fibre orientations) needs to be experimentally investigated



SN-curve and Goodman-diagram [FLEMMING, 2003]

Requirements of a New Analysis Concept Covering Fatigue of Composites

- Layer-wise approach:
  - ➢ generalized formulation (analysis of different laminate configurations)
  - ➢ more precise model description
- Less empirical, more physically motivated description of the material behaviour:
  - ➢ different failure modes, suitable failure criterion
  - ➢ failure mode dependent degradation of stiffnesses E_i^j
  - ➢ failure mode dependent degradation of strengths R_i^j
- Determining stress redistributions
  → larger structures or structural components need to be calculated efficiently
- Great number of loading cycles ($n \approx 10^9$) makes a cycle-by-cycle-analysis impossible
  → „Cycle-Jump"-strategy is pursued



Midsize rotorblade:

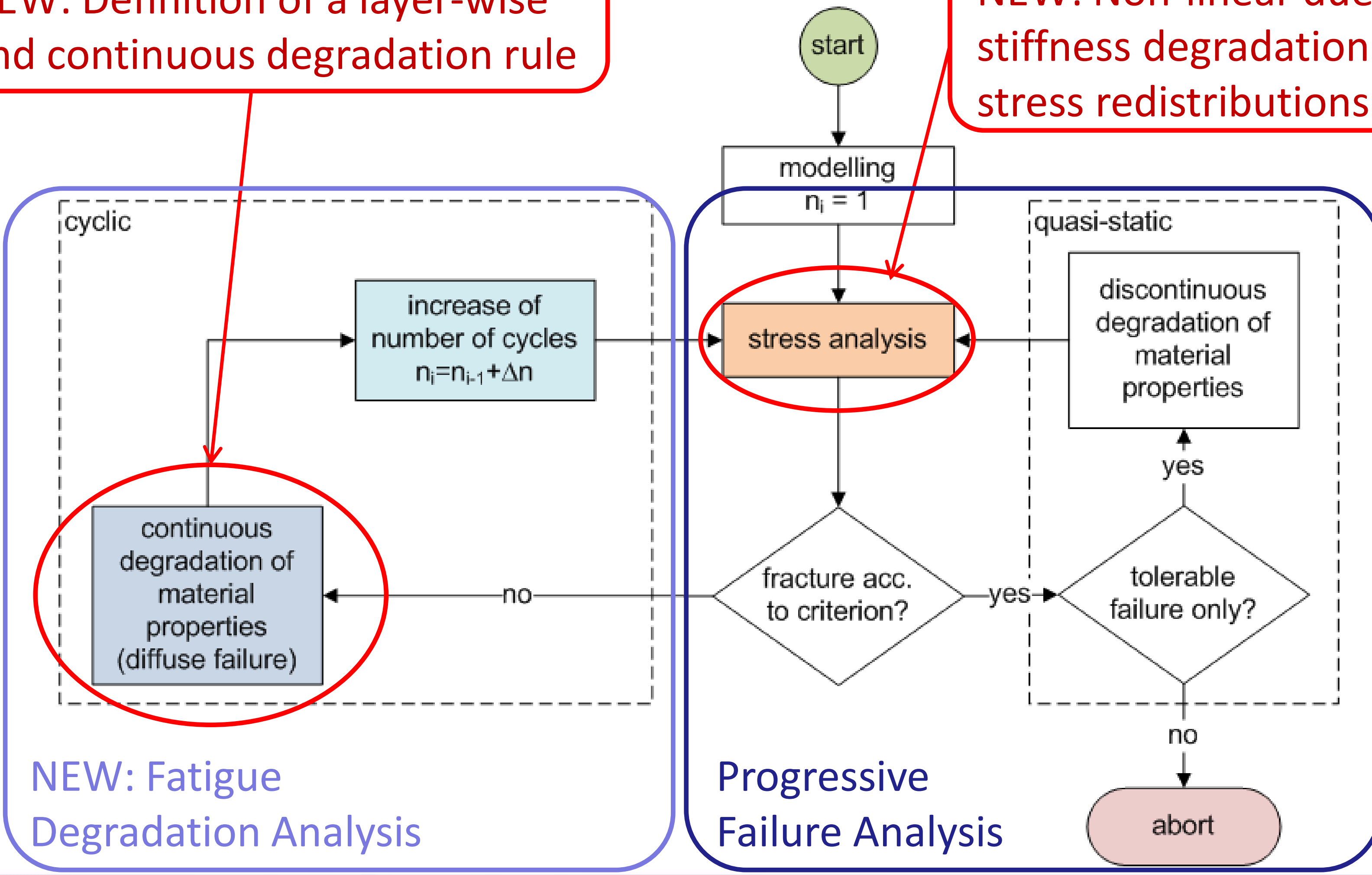
- ca. 130 different lay-ups

- up to ca. 270 plies

Overview:
Structural Analysis with Fatigue Evaluation

NEW: Definition of a layer-wise and continuous degradation rule

NEW: Non-linear due to stiffness degradation and stress redistributions



Overview:
Input and Output

Input:

- External Loadings

- associated number of cycles (n_i)

Analysis

Output (2D):

- stiffness degradation:
η_{E11}^{t}, η_{E11}^{c}, η_{E22}^{t}, η_{E22}^{c}, η_{E21} (or D_{Ei})

- strength degradation:
η_{R11}^{t}, η_{R11}^{c}, η_{R22}^{t}, η_{R22}^{c}, η_{R21} (or D_{Ri})

- fatigue related strains:
ε_i^{fat}

Degraded stiffness and strength:

$E^j_{i,D} = \eta^j_{Ei}\, E_i = (1 - D^j_{Ei})\, E_i$

$R^j_{i,D} = \eta^j_{Ri}\, R^j_i = (1 - D^j_{Ri})\, R^j_i$

Hypothesis – the Main Constituent

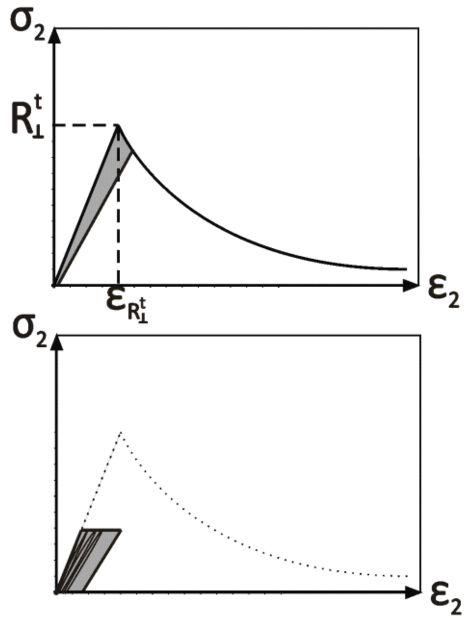
Energetic considerations for the determination of the layer-wise degradation

Hypothesis: „The damage state only depends on the amount of dissipated energy and the damage state, irrespective of how the structure has been loaded, is comparable in the sense of mechanical properties as stiffness and strength.“

Ex.: layer with unidirectional tensile loading perpendicular to fibre direction:

Quasi-static stress-strain-curve (in-situ)

Cyclic stress-strain-curve (in-situ)



$$W^{da}(D)$$

$$=$$

$$W^{fat}(D,\sigma^{fat},n)$$

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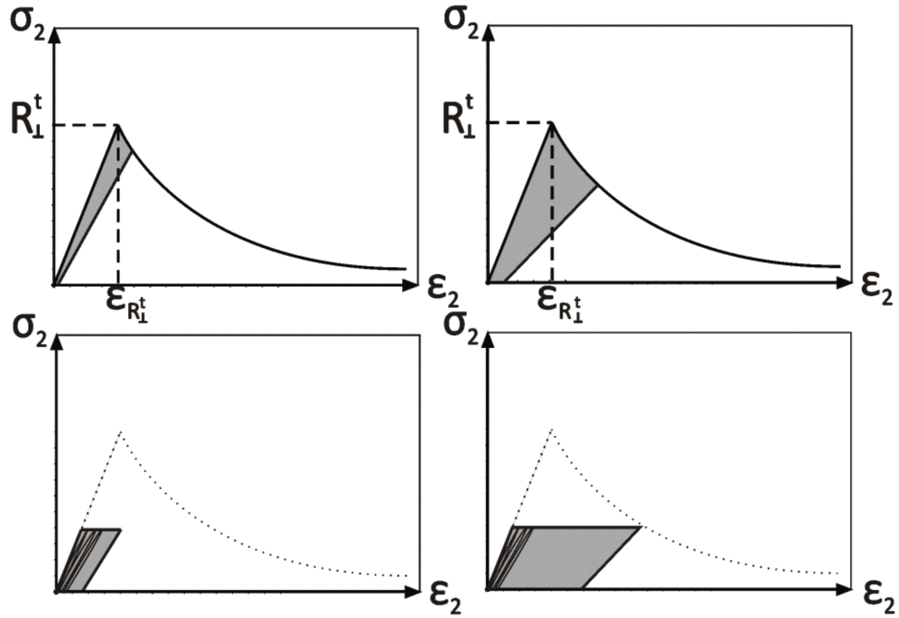

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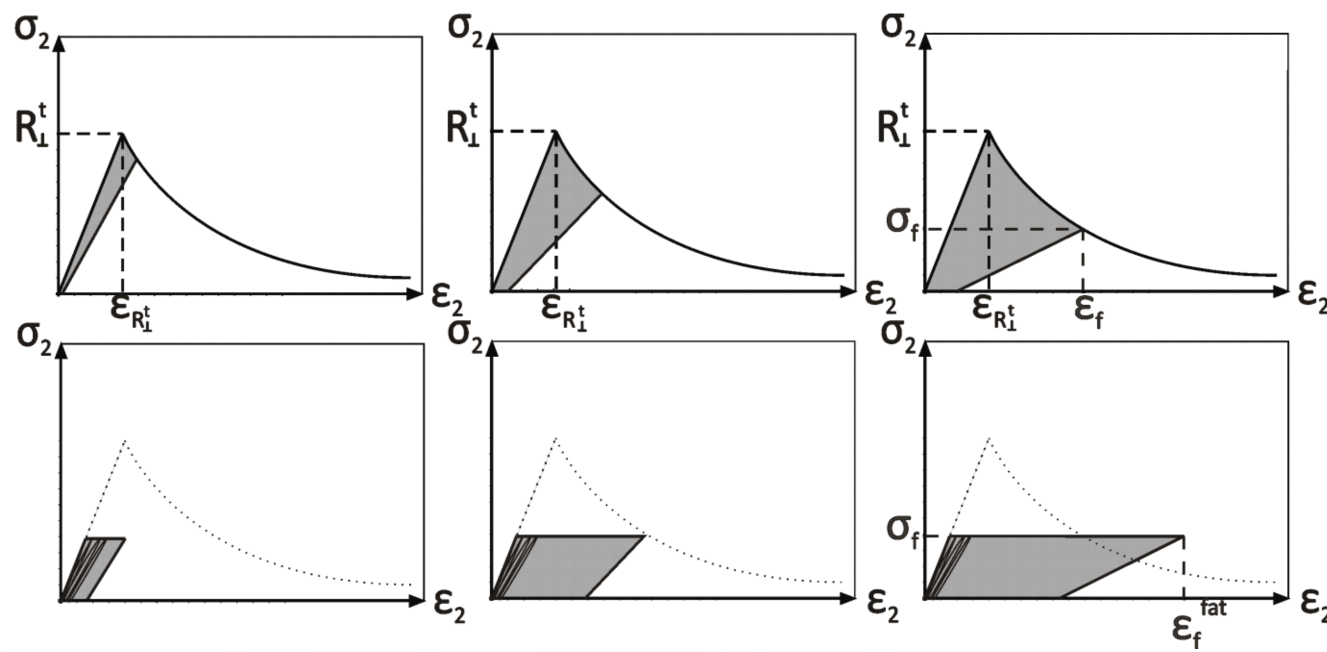

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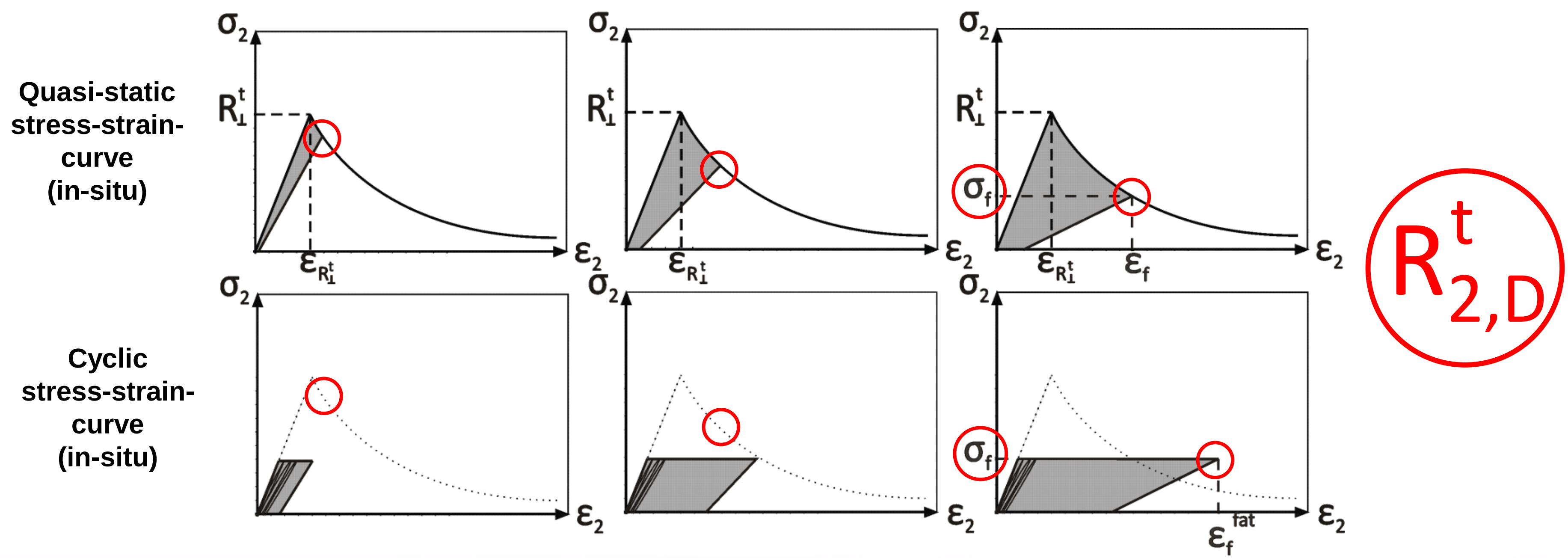

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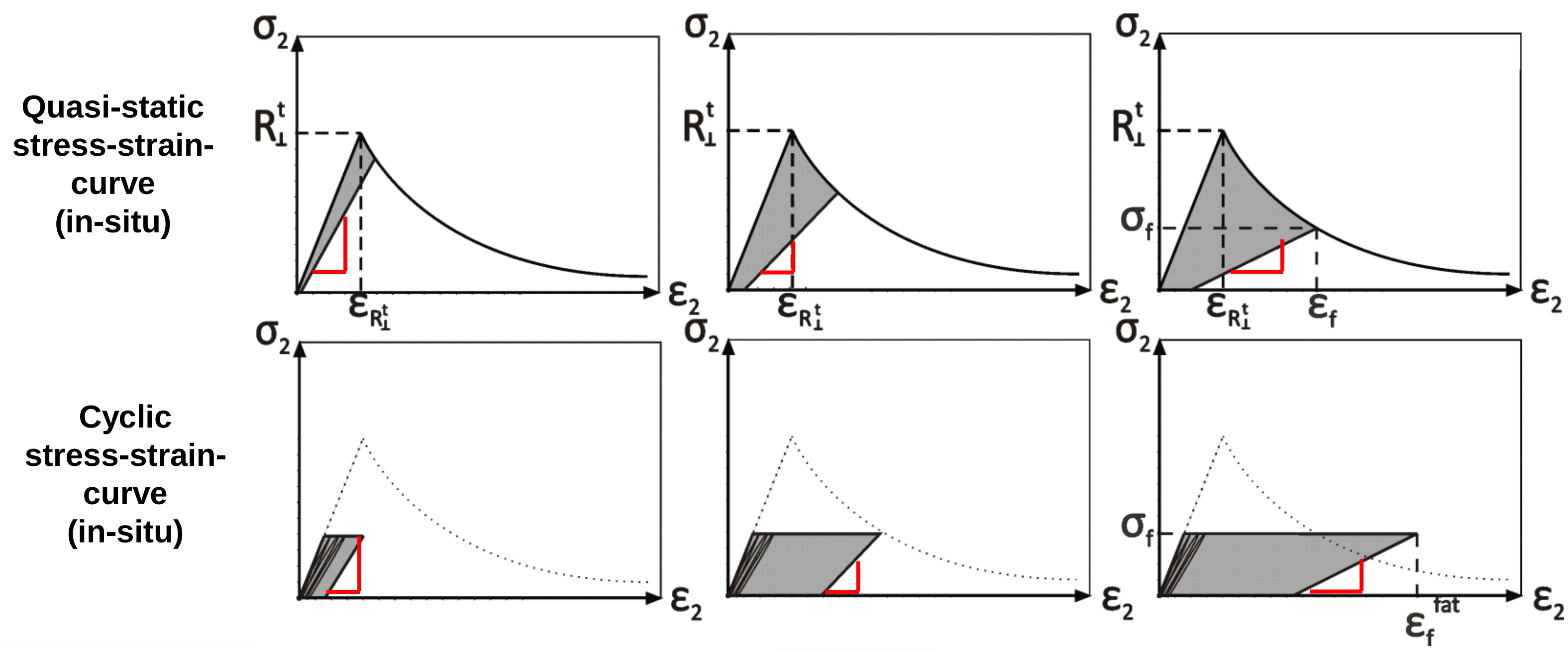

$E_{2,D}^t$

Testing the Concept: Numerical Example of an Open-Hole Panel

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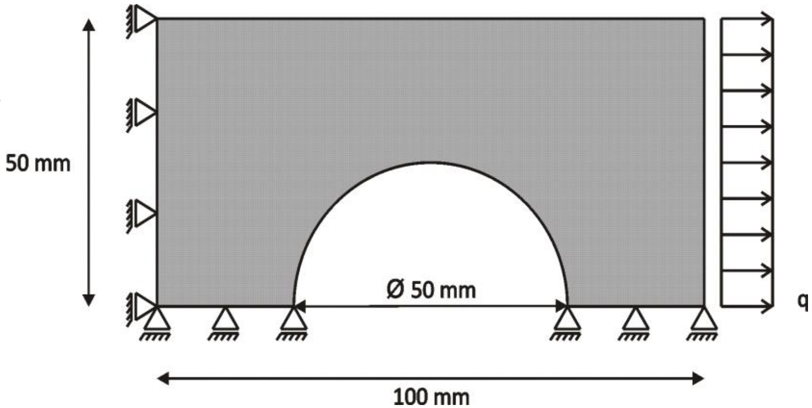
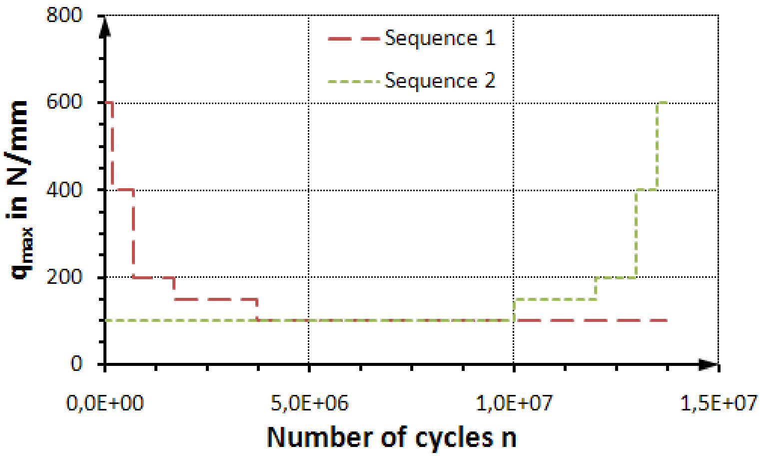
Influence of the loading sequence on the degradation factor $\eta_{E2}{}^{t}$ of a $[0/90]_S$-GFRP-laminate under horizontal tensile fatigue loading (R=0.1)

- Quadratic open-hole panel, simply supported, symmetric boundaries
- Layered shell elements
- Implementation of the concept as a material routine
- Horizontal tensile loading
- Two loading sequences, VA-block loading:
  - Sequence 1: decreasing
  - Sequence 2: increasing





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Testing the Concept:
Numerical Example of an Open-Hole Panel

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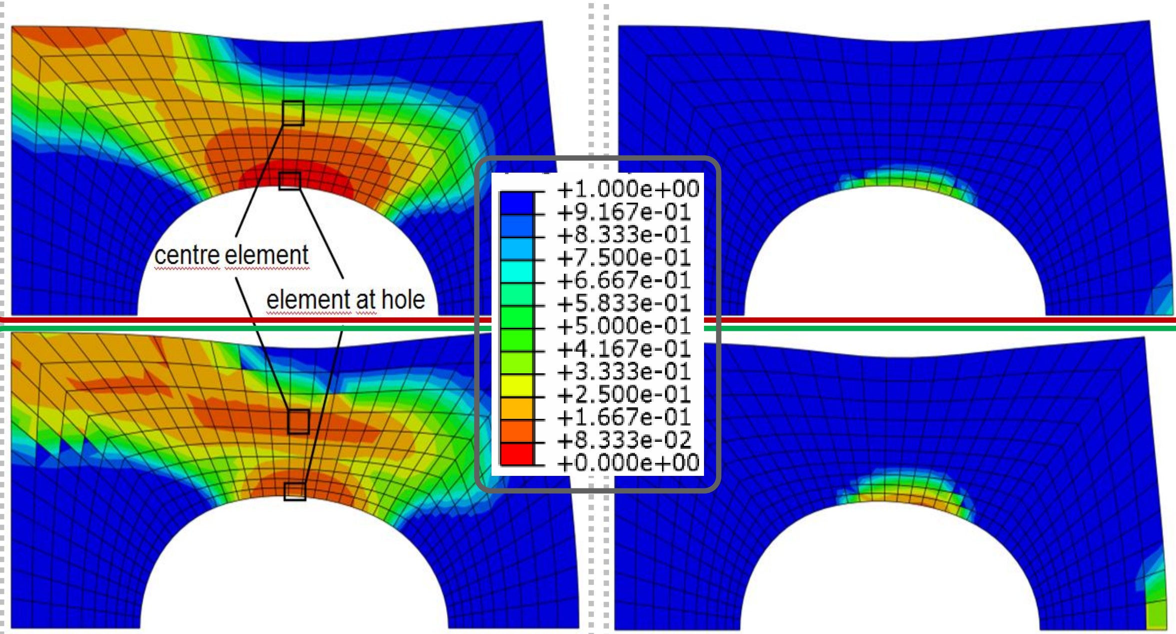
Influence of the loading sequence on the degradation factor η_{E2}^{t} of a $[0/90]_S$-GFRP-laminate under horizontal tensile fatigue loading (R=0.1)

90°-layer | 0°-layer

Decreasing loading sequence



Increasing loading sequence

Decreasing loading sequence:

→ small loads after high loading do not cause significant damage

→ more local and higher degradation

Increasing loading sequence:

→ small loads cause significant predamaging around the hole

→ more distributed and lower degradation

Conclusions

- The current-practice fatigue analysis procedure is not suitable for the complex material behaviour of FRPs.
- The fatigue concept proposed is able to overcome various shortcomings:
  - Non-linear damage accumulation
  - Differentiating of failure modes
  - Determination of the degradation is possible at each point of the fatigue history.
  - Discontinuous and continuous degradation of stiffness and strength allows for simulating stress redistributions and analysing sequence effects.
  - Due to the layer-based approach each arbitrary laminate set-up can be analysed.
- Testing the concept on an open-hole panel shows promising results.

Concluding Remarks

- The present work has shown that significant steps towards the availability of reliable, accurate, and economically efficient prediction methods can be made, by
  - looking "deeper" into the material,
  - using progressive failure analysis,
  - using efficient probabilistic methods.
- The challenges remain!

Acknowledgements

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State of the Art Simulation of Composite Structures

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