Optimization and Robustness Analysis in Structural Mechanics with LS-OPT

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About LS-OPT

- LS-OPT is a product of LSTC (Livermore Software Technology Corporation)
- LS-OPT can be linked to any simulation code – stand alone optimization software

Methodologies/Features:
- **Successive Response Surface Method (SRSM)**
- **Search Based optimization (SRS) – “moving clouds”**
- **Reliability based design optimization (RBDO)**
- **Multidisciplinary optimization (MDO)**
- **Multi-Objective optimization (Pareto)**
- **numerical/analytical sensitivities gradient based**
- **Analysis of Variance (ANOVA)**
- **Stochastic/Probabilistic Analysis**
- **Monte Carlo Analysis using Metamodules**
About LS-OPT

- LS-OPT provides a **graphical user interface (GUI)** – interaction with LS-PrePost
- Job Distribution - Interface to Queuing Systems
  - PBS, LSF, LoadLeveler, AQS
- LS-OPT might be used as a “Process Manager”
- Shape Optimization
  - Interface to HyperMorph, DEP-Morpher
  - User-defined interface to any Pre-Processor
## Design Variables

### Imported Variables

- **Variables in Keyword file automatically imported into GUI**
- **LS-DYNA *PARAMETER keyword support**
  
  *Parameter definitions automatically imported into GUI*

- **Include files recursively parsed for parameters/variables**
Design Variables

Type of Variables

- Variable – standard design variable
- Constant – fixed variable
- Dependent variable:

  Ex.: variable ‘Youngs_modulus’ 2.0e08
  variable ‘Poisson_ratio’ 0.42
  dependent ‘Shear Modulus’ {Youngs_modulus/(2*(1+Poisson_ratio))}

- Noise variable – stochastic analysis
Evaluation of Results (responses)

Definition of History Responses

Ex.: \( u_{12}(t) = u_1(t) - u_2(t) \) -> Relative Displacement between two Nodes

Mathematical Expressions

- All C language type expressions...
- Integrals, Derivatives, Lookup Functions, Min/Max...

Ex.: \( \text{Max}(u_{12}(t), 5, 45) \) -> Maximum between 5 and 45 ms

Comfortable extraction of LS-DYNA results within the GUI

- LS-DYNA ASCII and binary (d3plot, binout) databases
- Mass, FLD, Injury Coefficients (HIC, CSI), Thickness, Frequency…
Multidisciplinary Optimization

- Sharing of Variables
  - Each discipline is defined by own variable subset

- Mode Tracking (Eigenvalue analysis)
  - Mode shape is tracked according to selected mode

- Discipline-Specific Sampling/Response Surface
  - Crash: RSM (Response Surface Method - usually D-Optimal DOE)
  - Vibration: DSA (Design Sensitivity Analysis – numerical/analytical)

- Discipline-Specific Job Distribution
  - Memory requirements may be solver dependent
Why Response Surface Methodology?

- Gradient based methods
  - Local Sensitivities may lead to local optima (highly nonlinear problems)
  - Difficulties by the Computation of Numerical Gradients

- Response Surfaces
  - Local minima caused by noisy response as well as the step-size dilemma for numerical gradients are avoided
Methods - Optimization

Optimization Process - Response Surface Methodology

- Objective
- Design Variable 1
- Design Variable 2
- Experimental Design points
- Subregion (Range)
- Starting (base) design
- Response surface
- Response values

- Design space

- Methods - Optimization
- Methods - Robustness
- Example I - Optimization
- Example II - Optimization
- Example III - Optimization
- Example I - Robustness
- Example II - Robustness

Outlook Version 3.0/3.1
Find an Optimum on the Response Surface (one iteration)

- Optimization of sub-problem (response surface) using LFOPC algorithm
- Starting value on response surface
- Optimum (computed by simulation using design variables)
- Optimum (predicted by response surface)
Successive Response Surface Methodology

- Design Space
- Region of Interest
- Optimum
- Start
- Design Variable 1
- Design Variable 2

Bounds of Region of Interest
Successive Response Surface Methodology

Example - 4th order polynomial

\[ g(x) = 4 + \frac{9}{2} x_1 - 4x_2 + x_1^2 + 2x_2^2 - 2x_1x_2 + x_1^4 - 2x_1^2x_2 \]
Successive Response Surface Methodology

Example - 4th order polynomial

\[ g(x) = 4 + \frac{9}{2}x_1 - 4x_2 + x_1^2 + 2x_2^2 - 2x_1x_2 + x_1^4 - 2x_1^2x_2 \]
Feasible Experimental Design

- Design Variable 1
- Design Variable 2
- Constraint $g$
- Constraint $f$
- Feasible Region
- Region of Interest
- Basis Experiments
- Center of Region of Interest (Baseline Design)
Design of Experiments (DOE) - Sampling Point Selection

- Koshal, Central Composite, Full Factorial
- D-Optimality Criterion - Gives maximal confidence in the model

\[
\max \left| X^T X \right|
\]

- Monte Carlo Sampling
- Latin Hypercube Sampling (stratified Monte Carlo)
- Space Filling Designs
Response Surfaces (Meta Models)
- Linear, Quadratic and Mixed polynomial based
- Neural Network and Kriging for Nonlinear Regression
Neural Network Regression

Example - 4th order polynomial

\[ g(x) = 4 + \frac{9}{2} x_1 - 4x_2 + x_1^2 + 2x_2^2 - 2x_1x_2 + x_1^4 - 2x_1^2x_2 \]

- analytical function (green)
- global neural net approx. with 20 points (red)
- simulation points
Successive Scheme with Neural Network

Example - 4th order polynomial

\[ g(x) = 4 + \frac{9}{2} x_1 - 4 x_2 + x_1^2 + 2 x_2^2 - 2 x_1 x_2 + x_1^4 - 2 x_1^2 x_2 \]
Error Analysis

- Meta Model Accuracy
- Error Analysis
  - RMS
  - Average error
  - Maximum error
  - PRESS (Prediction Error)
  - R2 indicator
Response Surface Based Variable Screening using ANOVA

Variable Screening

- **ANOVA – Analysis of Variance**
- **Removal of unimportant variables**
- **Confidence levels of each variable**

![Significance of Variable](image)

![Confidence Interval](image)
Response Surface Based Variable Screening

- $\Delta b_j$ depends on the variance of the simulation points
- Use a 90% confidence level and determine the lower bound
- Variables are ranked according to lower bound

From regression analysis –
Coefficient of variable $j$

Significant

Insignificant

Value which determines significance
Multi-Objective Optimization

Simple Example: Cantilever Beam

Design Objective
- Minimize truss volume (mass)
- Minimize maxStress

Design variables
- Cross section area $A$
- Angel $\alpha$

Design space
- $A \in [10mm^2; 100mm^2]$  
- $\alpha \in [5^\circ; 85^\circ]$
Multi-Objective Optimization - Volume vs. Stress

Trade-Off Study using Neural Network Response Surface
### Optimization Methodologies for highly nonlinear Applications

<table>
<thead>
<tr>
<th>Gradient based methods</th>
<th>Random Search</th>
<th>Evolutionary Algorithms</th>
<th>RSM / SRSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>▪ accuracy of solution</td>
<td>▪ very robust, can not diverge</td>
<td>▪ good for problems with many local minimas</td>
<td>▪ very effective, particularly SRSM</td>
</tr>
<tr>
<td>▪ number of solver calls</td>
<td>▪ easy to apply</td>
<td></td>
<td>▪ trade-off studies on RS</td>
</tr>
<tr>
<td>▪ can diverge</td>
<td>▪ bad convergence, not effective</td>
<td>▪ many solver calls, only suitable for fast solver runs</td>
<td>▪ filter out noise, smoothing of results</td>
</tr>
<tr>
<td>▪ can stuck in local minimas</td>
<td>▪ Chooses best observation – may not be representative of a good (robust) design</td>
<td>▪ Chooses best observation – may not be representative of a good (robust) design</td>
<td></td>
</tr>
<tr>
<td>▪ step-size dilemma for numerical gradients</td>
<td></td>
<td></td>
<td>▪ approximation error, verification run may be infeasible</td>
</tr>
</tbody>
</table>
Methods – Robustness

Stochastic/Probabilistic Analysis

Varying Input Parameters
(test setup, tolerances)

CAE-Simulation

Varying Output Parameters

System Inherent Variations
(Bifurcations, Chaos, Unpredictable Variations)
Methods – Robustness

- Stochastic/Probabilistic Analysis
  - Statistical Distributions
    - Beta
    - Binomial
    - Lognormal
    - Normal
    - Uniform
    - User defined
    - Weibull
  - Response Variability
    - Response distribution,
    - Mean, Standard deviation
    - Probability of Failure
Methods – Robustness

➡ Stochastic/Probabilistic Analysis

- Monte Carlo
  - *Latin Hypercube sampling*
  - *Large number of FE runs (100+)*
  - *Random process*

- Monte Carlo using Meta-Models
  - *Response Surface / Neural Network*
  - *Medium number of FE runs (10 – 30+)*
  - *Identify design variable contributions clearly*

- Outlier investigation
  - *Unexpected events*
Meta-Modeling and Stochastic Contributions

\[ \sigma_T^2 = \sigma_R^2 + \sigma_D^2 \]

- Stochastic Contributions

- Noisy Physical Response
- Finite Element Simulations
- Meta-Model (Least-Squares Fit)

- Design Variable
- \( \sigma_{Var} \)
- \( \sigma_{Var} \)
Test vs. Analysis – Example Front Crash

\[ \Delta_{\text{exp-test}} = 15\% \]

Crash Test

Standard Deviation of Main Rail Yield

Expected Range for Maximum Steering Wheel x intrusion

95% probability

67% probability

Expected value

\[
\begin{align*}
\text{Expected value} & = \text{Expected Range for Maximum Steering Wheel x intrusion} \\
\text{95\% probability} & = \text{Upper limit} \\
\text{67\% probability} & = \text{Middle limit} \\
\Delta_{\text{exp-test}} & = \text{15\%} \\
\text{Expected value} & = \text{20\%}
\end{align*}
\]
Test vs. Analysis

- Simulation Mean
- A Simulated Item
- Systematic Uncertainty
- Actual Mean
- An Actual Item
- Random Uncertainty
- Actual Safety Margin
- Simulated Safety Margin

Failure
Reliability Analysis

- Probability of failure
- Evaluation of confidence interval
- Prediction error (confidence interval) depends
  - on the number of runs
  - on the probability of event
  - not on the dimension of the problem (number of design variables)
Buckling Analysis - Fringe Components of Displ/Velo/Accl-Variance (40 runs)

- Standard deviation of y-displacements of each node

- High Variance of y-displacement

RUN 1
Buckling mode A

RUN 8
Buckling mode B

Courtesy DaimlerChrysler
Parameter Identification of Plastic Material

- Material properties: nonlinear visco-elastic behaviour
- LS-DYNA hyperelastic/viscoelastic formulation - *MAT_OGDEN_RUBBER (#77)
- Hyperelasticity

\[ W = \sum_{i=1}^{3} \sum_{j=1}^{n} \frac{\mu_j}{\alpha_j} (\lambda_i^\alpha_j - 1) + \frac{1}{2} K (J - 1)^2 \]

- Prony series representing the visco-elastic part (Maxwell elements):

\[ g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t} \quad ; \quad N=1, 2, 3, 4, 5, 6 \quad ; \quad \sigma_{ij} = \int_{0}^{t} g_{ijkl}(t-\tau) \frac{\partial \epsilon_{kl}}{\partial \tau} d\tau \]
Parameter Identification of Plastic Material

- Minimize the distance between experimental curve and simulation curve
- Least-Squares Objective Function

\[
F(x) = \sum_{p=1}^{P} \left\{ \left[ y(x) - f(x) \right]^2 \right\} \rightarrow \min F(x)
\]
Example II – Optimization

Shape Optimization of a Crash Box

Scope of optimization:

- minimize the maximum crash force
- steady-going force progression

Shape variation by using Hypermorph and LS-OPT (20 design variables)
Optimization of a Van Component Model

Scope of optimization: Assembly of a vehicle body for a commercial van
Optimization of a Van Component Model

- Load is applied displacement driven by a constant velocity of the stonewall in x-direction

- Monitored responses:
  - Internal Energy of components
  - Stonewall Force
Example III – Optimization

Optimization Problem

Objective

- Maximize the ratio of the maximum value of the internal energy and the mass of the considered components

\[ E_M = \frac{E_{\text{max}}}{M} \]

Constraint

- Upper Bound for the stonewall force

\[ \text{maxRWFORCE} < 1.25 \]

Design Variables

- Sheet thicknesses of 15 parts
- Beads defined by 5 design variables
Example III – Optimization

First Stage

- Latin Hypercube Sampling with 20 variables

- Random search based design improvement (3 iterations) with in total 150 runs
  
  - 22% design improvement

- Performing ANOVA analysis in order to reduce the number of design variables
Example III – Optimization

- Second Stage
  - Deterministic Optimization using the Successive Response Surface Method (SRSM) with the 4 most significant variables
  - Starting values are taken out of the best run of the 150 random simulations (First Stage)

- Additional 10% design improvement
Example I – Robustness

Robustness Investigations – Monte Carlo analysis

- Variation of sheet thicknesses and yield stress of significant parts in order to consider uncertainties
- Normal distribution is assumed
  - $T_{1134}$ (Longitudinal Member) $\text{mean} = 2.5\text{mm}; \sigma = 0.05\text{mm}$
  - $T_{1139}$ (Closing Panel) $\text{mean} = 2.4\text{mm}; \sigma = 0.05\text{mm}$
  - $T_{1210}$ (Absorbing Box) $\text{mean} = 0.8\text{mm}; \sigma = 0.05\text{mm}$
  - $T_{1221}$ (Absorbing Box) $\text{mean} = 1.0\text{mm}; \sigma = 0.05\text{mm}$
  - $SF_{1134}$ (Longitudinal Member) $\text{mean} = 1.0; \sigma = 0.05$

- Monte Carlo analysis using 182 points (Latin Hypercube)
Example I – Robustness

Tradeoff Plot

- Monte Carlo Simulation
- Identification of Clustering

Simulation 185 folding

Simulation 47 buckling

Monte Carlo Simulation

Simulation 47 buckling

internal energy

sheet thickness $T_{1139}$
Example I – Robustness

→ Reliability Analysis
- Histogram of distribution
- Probability of exceeding a constraint-bound

→ Min-Max Curves
- Plot of minimum, maximum and mean history values
- Gives a confidence interval of history values

Probability of 8.4% for violating the RWFORC-bound

Min-Max Curves

Optimization/Robustness Analysis in Structural Mechanics – 07.06.05
Example II – Robustness

Design Variables - Uncertainties in Test Set-Up

- Slip Ring Friction: `sfric1`
- Airbag Mass Flow: `scal_massflow`
- Pre-Tensioner: `preten`
- Steering Wheel: `rot_stwh`
- Force Limit Retractor: `forcelimit`
- Dashboard: `young_alu x_transl z_transl`
- Sled Acceleration: `scalaccel`
- Slip Ring Friction: `sfric2`
Example II – Robustness

Responses: Standard Dummy Evaluations

Head Impact Criterion
HIC36

Chest Intrusion
max_chest_intru

Chest Acceleration
max_chest

Belt Force Shoulder
max_belt_force_shoulder

Belt Force Pelvis
max_belt_force_pelvis

Pelvis Acceleration
max_pelvis
### Example II – Robustness

#### Stochastic Contribution - Results of 30 Experiments

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Standard Deviation of Design Variable</th>
<th>Standard Deviation Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>scalaccel</td>
<td>2.5%</td>
<td>3.1% 1.5% 0.1% 2.3% 1.9% 2.9%</td>
</tr>
<tr>
<td>sfric1</td>
<td>25.0%</td>
<td>1.3% 0.6% 4.1% 1.8% 0.7% 0.7%</td>
</tr>
<tr>
<td>sfric2</td>
<td>25.0%</td>
<td>0.5% 0.6% 0.1% 3.7% 0.1% 0.1%</td>
</tr>
<tr>
<td>preten</td>
<td>4.4%</td>
<td>0.0% 0.5% 0.0% 1.1% 0.3% 0.2%</td>
</tr>
<tr>
<td>forcelimit</td>
<td>5.6%</td>
<td>1.3% 0.4% 4.4% 0.6% 1.4% 0.2%</td>
</tr>
<tr>
<td>rot_stwh</td>
<td>4.8%</td>
<td>0.5% 0.1% 0.1% 0.0% 0.1% 0.1%</td>
</tr>
<tr>
<td>transl_x</td>
<td>50.0%</td>
<td>0.1% 0.1% 0.7% 4.5% 0.5% 0.8%</td>
</tr>
<tr>
<td>transl_z</td>
<td>50.0%</td>
<td>1.2% 1.0% 0.3% 1.6% 0.2% 0.9%</td>
</tr>
<tr>
<td>scalmassflow</td>
<td>5.0%</td>
<td>1.8% 1.8% 0.6% 2.2% 0.6% 0.9%</td>
</tr>
<tr>
<td>young_alu</td>
<td>5.0%</td>
<td>0.3% 0.3% 0.0% 0.5% 0.1% 0.1%</td>
</tr>
<tr>
<td>all variables</td>
<td></td>
<td>4.3% 2.8% 6.1% 7.2% 2.6% 3.4%</td>
</tr>
<tr>
<td>residuals</td>
<td></td>
<td>4.7% 1.9% 1.8% 6.0% 3.5% 2.3%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>6.4% 3.4% 6.3% 9.4% 4.3% 4.1%</td>
</tr>
</tbody>
</table>

**Contribution of variation of design variables to variation of results**

- Meta-model space
- Residual space
- Total Variation
Example II – Robustness

Standard deviation of x-displacements of each node (120 runs)

- Deterministic (Meta-Model)
- Residual (Outliers)
Version 3.0 - Announced 4th Quarter 2005

- LS-OPT for Windows
  - Incorporates new Application Program Interface to speed up development/facilitate porting
- Parameter Identification Module (beta available)
  - Automated use of test results to calibrate materials/systems
  - Simplify input for system identification applications
  - Handles "continuous" test curves
- Improved visualization of stochastic results
  - Extended LS-PREPOST visualization of design sensitivities and importance of design variables
- Reliability-based design optimization (RBDO)
  - Specify probability of failure as design constraints
Version 3.1 - 2006

- Discrete Optimization
  - Define fixed sets for variables
  - Discrete materials (combinatorial problem)
- 3-D visualization of response surfaces
  - OpenGL interface
- GUI features for expressions and special composite functions, e.g. parameter identification, integration, …
- …
Thanks for your attention!