Material Models of Polymers for Crash Simulation

An overview with focus on the dynamic test setup Impetus by 4a engineering

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Dynamore Infotag "Impetus", 30.11.2009
Labotatory of Mechanics - Equipment

- Hardware / Software
  - Clusters of Xeon, Intel Dual-Core and Quad-Core, 8CPUs parallel
  - FE Packages: LS-Dyna, Radioss, Nastran
  - Pre and Postprocessor: Hyperworks, LS-PrePost

- Experimental Setups
  - Quasi-static tensile and compression tester by Instron
  - Dynamic testing system “4a Impetus II”, movable devices for compression and bending tests, range of velocities: 500-4500mm/s
  - Dynamic test setup for impact tests on windshields
  - Drop tower
Material Models of Polymers for Crash Simulation

Outline

- Parameter based Input vs. Tabulated Input
- Rubberlike Materials
  - Finite Elasticity
  - Blatz-Ko Rubber (Mat_7)
  - Simplified Rubber (Mat_181)
- Foams
  - Fu Chang Foam (Mat_83)
  - Simplified Rubber (Mat_181)
- Plastics
  - Piecewise Linear Plasticity Mat_24
  - Schmachtenberg / Johnson Cook
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Input of stress-strain relations in a tabulated way are very popular in commercial crash-codes.

The (more or less) direct input of experimental data obtained by tensile tests is the major benefit of those approaches.

This advantage fails in the validation and verification process where the stress-strain-curves have to be fitted to experimental results.

Parameter based stress-strain relations have therefore a huge advantage in reverse engineering (fitting of parameters, e.g. by LS-OPT, instead of the entire stress-strain-datapoints).
- **Parametrized Formulation**

![Graph showing stress-strain relationship](image)

\[ \sigma(\varepsilon, a_i) \]

- Usually via suitable ansatz \( \sigma(\varepsilon, a_i) \) in dependence of the material under consideration, where \( a_i \) are material parameters.
Parameters may then be identified, e.g. by least square fit:

\[
S(a_i) := \sum_{k=1}^{n} \left[ \sigma_k (\varepsilon_k) - \sigma(\varepsilon, a_i) \right]^2 \rightarrow \text{MIN}
\]

\[
\Rightarrow \quad \partial_{a_1} S(a_1) = \partial_{a_2} S(a_2) = \ldots = \partial_{a_n} S(a_n) = 0
\]

which leads to a nonlinear system of equations in general

- Alternatively LS-OPT can also be used
Parameter based Input versus Tabulated Input

- Procedures of Material Card Generation

1. Experimental Data
2. Parametrization
3. Fitting Loop
4. Optimized Parameters
5. Save Curve to Data Points
6. Material Law with Tabulated Input
7. Parameter based Material Law
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Hyperelasticity

Right Cauchy-Green Tensor \( C = F^T F \) with \( F = \text{Grad} x \)

2. PK: \( S = 2 \frac{\partial W}{\partial C} \rightarrow \sigma = \frac{1}{J} F S F^T \) \( J = \det F \)

Cauchy stress tensor

Strain energy density in terms of invariants: \( W = \hat{W}(I_C, II_C, III_C) \)

\[
I_C = 1: C = \text{tr} C, \quad II_C = \frac{1}{2} \left( I_C^2 - C : C \right), \quad III_C = \det C
\]

Derivative:

\[
S = 2 \frac{\partial W}{\partial I_C} 1 + 2 \frac{\partial W}{\partial II_C} (I_C 1 - C) + 2 \frac{\partial W}{\partial III_C} III_C C^{-1}
\]
## Rubber Laws in LS-DYNA

<table>
<thead>
<tr>
<th>Law</th>
<th>Law Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>MAT_BLATZ-KO_RUBBER</td>
</tr>
<tr>
<td>27</td>
<td>MAT_MOONEY-RIVLIN_RUBBER</td>
</tr>
<tr>
<td>31</td>
<td>MAT_FRAZER-NASH_RUBBER</td>
</tr>
<tr>
<td>77</td>
<td>MAT_GENERALIZED_RUBBER</td>
</tr>
<tr>
<td>77</td>
<td>MAT_OGDEN_RUBBER</td>
</tr>
<tr>
<td>181</td>
<td>MAT_SIMPLIFIED_RUBBER</td>
</tr>
</tbody>
</table>
One-Parameter Law: Blatz-Ko Energy Function

- General form for polyurethane foam rubbers (1962):

\[
W = \frac{G}{2} \left[ I_1 + \frac{1}{\alpha} \left( I_3^{-\alpha} - 1 \right) - 3 \right] + \frac{G}{2} (1 - \beta) \left[ \frac{I_2}{I_3} + \frac{1}{\alpha} \left( I_3^\alpha - 1 \right) - 3 \right]
\]

\[
\alpha = \frac{\nu}{1 - 2\nu}
\]

- Implemented as material law no. 7 in LS-DYNA:

\[
\beta = 1, \quad \nu = 0.463
\]

\[
W = \frac{G}{2} \left[ I_1 - 3 + \frac{1}{\alpha} \left( I_3^{-\alpha} - 1 \right) \right] \quad \Rightarrow \quad \sigma = G \left( \frac{1}{J} \mathbf{F F}^T - J^{-2\alpha - 1} \delta \right)
\]

\[
\alpha = \frac{\nu}{1 - 2\nu} \quad \Rightarrow \quad -2\alpha - 1 = -2 \frac{\nu}{1 - 2\nu} - 1 = -\frac{1}{1 - 2\nu}
\]
### Equivalent One-Parameter Models

<table>
<thead>
<tr>
<th>7</th>
<th>27</th>
<th>77 Ogden</th>
<th>31</th>
<th>77</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>$A = \frac{G}{2}$</td>
<td>$\mu_1 = G$</td>
<td>$C_{100} = \frac{G}{2}$</td>
<td>$C_{10} = \frac{G}{2}$</td>
</tr>
</tbody>
</table>
Equivalent One-Parameter Models

![Graph showing one parameter rubber models with different stress-strain curves for various materials.](image)

- A. Blatz–Ko tension
- B. Blatz–Ko compression
- C. Mooney–Rivlin tension
- D. Mooney–Rivlin compression
- E. Ogden tension
- F. Ogden compression
- G. Frazer–Nash tension
- H. Frazer–Nash compression
- I. Hyperelastic tension
- J. Hyperelastic compression
Material Models of Polymers for Crash Simulation

**Limitations of Low Order Models**

- Fitting of a higher curvature in the stress-strain curve for large deformations will not work
- Optimization software will not help
- Multiple parameter models, e.g. Ogden’s energy function (Mat_77) allow for fitting stress-strain curves with higher curvature

\[ W = \sum_{i=1}^{3} \sum_{j=1}^{n} \frac{\mu_j}{\alpha_j} \left( \lambda_i^{*\alpha_j} - 1 \right) + K \left( J - 1 - \ln J \right) \]

\[ J = \lambda_1 \lambda_2 \lambda_3, \quad \lambda_i^{*} = \lambda_i J^{-1/3} = \frac{\lambda_i}{J^{1/3}} \]

- Tabulated version available in MAT_SIMPLIFIED_RUBBER
## Equivalent Multiple-Parameter Models

<table>
<thead>
<tr>
<th>31</th>
<th>77</th>
</tr>
</thead>
<tbody>
<tr>
<td>C100</td>
<td>$I_1$</td>
</tr>
<tr>
<td>C200</td>
<td>$I_1^2$</td>
</tr>
<tr>
<td>C300</td>
<td>$I_1^3$</td>
</tr>
<tr>
<td>C400</td>
<td>$I_1^4$</td>
</tr>
<tr>
<td>C110</td>
<td>$I_1 I_2$</td>
</tr>
<tr>
<td>C210</td>
<td>$I_1^2 I_2$</td>
</tr>
<tr>
<td>C010</td>
<td>$I_2$</td>
</tr>
<tr>
<td>C020</td>
<td>$I_2^2$</td>
</tr>
</tbody>
</table>
Equivalent Multiple-Parameter Models

6 parameter rubber models

- A: Frazer–Nash tension
- B: Frazer–Nash compression
- C: Hyperelastic tension
- D: Hyperelastic compression

Z-stress in GPa

Time

Element No
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Dynamore Infotag Impetus, 30.11.2009
Introduction - What are Foams?

- Material scientist: any material manufactured by some expansion process
- (Crash-) Numericist: a material with Poisson’s ratio close to zero

- Both definitions coincide only for low density foams, roughly below 200g/l
- High density (>200g/l) structural foams exhibit a non-negligible Poisson effect
## Material Laws for Elastic Foams in LS-DYNA

<table>
<thead>
<tr>
<th>No.</th>
<th>keyword</th>
<th>formulation</th>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>MAT_BLATZ_KO_FOAM</td>
<td>hyperel., $\nu = 0.25$</td>
<td>1 parameter</td>
</tr>
<tr>
<td>57</td>
<td>MAT_LOW_DENSITY_FOAM</td>
<td>hyperel. + viscoel.</td>
<td>LC + parameter</td>
</tr>
<tr>
<td>62</td>
<td>MAT_VISCOUS_FOAM</td>
<td>hyperel. + viscoel.</td>
<td>parameter</td>
</tr>
<tr>
<td>73</td>
<td>MAT_LOW_DENSITY_VISCOUS_FOAM</td>
<td>hyperel. + 6 viscoel. dampers</td>
<td>LC + parameter</td>
</tr>
<tr>
<td>83</td>
<td>MAT_FU-CHANG_FOAM</td>
<td>hyperel. + strain-rate</td>
<td>LC/ table</td>
</tr>
<tr>
<td>177</td>
<td>MAT_HILL_FOAM</td>
<td>hyperel., $\nu$ variable</td>
<td>LC</td>
</tr>
<tr>
<td>178</td>
<td>MAT_VISCOELASTIC_HILL_FOAM</td>
<td>= 177 + viscoel</td>
<td>LC + parameter</td>
</tr>
<tr>
<td>179</td>
<td>MAT_LOW_DENSITY_SYNTETIC_FOAM</td>
<td>hyperel.</td>
<td>LC</td>
</tr>
<tr>
<td></td>
<td></td>
<td>pseudo-damage</td>
<td>LC</td>
</tr>
<tr>
<td>180</td>
<td>MAT_LOW_DENSITY_SYNTETIC_FOAM ORTHO</td>
<td>no damage orthog-onal load direction</td>
<td>LC</td>
</tr>
<tr>
<td>181</td>
<td>MAT_SIMPLIFIED_RUBBER/FOAM (WITH_FAILURE) / WITH_DAMAGE</td>
<td>hyperel. + strain-rate $\nu$ variable</td>
<td>LC/ table</td>
</tr>
</tbody>
</table>


Material Models of Polymers for Crash Simulation

Material Laws for Elastic Foams (no Poisson Effect)

strainrate dependent hyperelastic

visco-hyperelastic

\[ \sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} \]

elastic damage

\[ \dot{\varepsilon}_2 > \dot{\varepsilon}_1 \]

\[ \dot{\varepsilon}_1 > \dot{\varepsilon}_0 \]

\[ \dot{\varepsilon}_0 = 0 \]

visco-hyperelastic

\[ \sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} \]
Material Laws for Elastic Foams (no Poisson Effect)

strainrate dependent hyperelastic

\[ \sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} \]

visco-hyperelastic

\[ \sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} \]

\[ \dot{\varepsilon}_2 > \dot{\varepsilon}_1 \]

\[ \dot{\varepsilon}_1 > \dot{\varepsilon}_0 \]

\[ \dot{\varepsilon}_0 = 0 \]

MAT_83
MAT_FU-CHANG_FOAM

MAT_57
MAT_LOW_DENSITY_FOAM

MAT_62
MAT_LOW_DENSITY_FOAM
Rate-Dependent Hyperelasticity versus Visco-Elasticity

Relaxation Test

- rate-independent unloading
- numerically stable
- unrealistic, potential problem for foams with high damping (e.g. confor foam)

\[ \dot{\varepsilon}_2 > \dot{\varepsilon}_1 \]
\[ \dot{\varepsilon}_1 > \dot{\varepsilon}_0 \]
\[ \dot{\varepsilon}_0 = 0 \]

\[ u = \varepsilon_0 \]

\[ \sigma \]

\[ \varepsilon \]

\[ t \]

strain-rate dependent elastic visco-elastic

MAT_83
MAT_73
Define an additional curve for unloading (strain rate zero in TABLE), this should correspond to the quasistatic unloading path.

Unloading always follows the curve with lowest strain rate and is detected by

$$\varepsilon_i \cdot \dot{\varepsilon}_i \begin{cases} 
\leq 0 & \rightarrow \text{unloading: strain rate is set to zero} \\
> 0 & \rightarrow \text{loading: strain rate dependence}
\end{cases}$$

This may lead to numerical problems that can be avoided by an elastic damage formulation.

Furthermore, no rate dependency upon unloading.
Some Validation Tests – How Accurate is MAT_83?

EPP RG30 90% compression

EPP RG40 tension

- RG40 test 500 mm/s
- RG40 test 5 mm/s
- MAT83 500 mm/s
- MAT83 5 mm/s
Some Validation Tests – How Accurate is MAT_83?

Material Models of Polymers for Crash Simulation
Some Validation Tests – How Accurate is MAT_83?

- Damage formulation a further improvement and can also be identified during the Impetus/LS-OPT procedure!
Material Law for Elastic Foams with Poisson Effect

- Uses Hill instead of Ogden functional (incompressible case, rubber):

\[
W = \sum_{j=1}^{m} \frac{C_j}{b_j} \left[ \lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n} (J^{-nb_j} - 1) \right]
\]

where \( C_j \), \( b_j \) and \( n \) are material constants and \( J = \lambda_1 \lambda_2 \lambda_3 \).

The nominal stresses (force per unit undeformed area) are

\[
S_i = \frac{1}{\lambda_i} \sum_{j=1}^{m} C_j \left[ \lambda_1^{b_j} - J^{-nb_j} \right] \quad i = 1, 2, 3
\]

- Allows for a fully tabulated input implemented as MAT_SIMPLIFIED_RUBBER/FOAM in 2004
Example: Rubberlike Foam for Sensomotoric Inlays

- In pendulum impact tests (Impetus) stress can be plotted as a function of strain and the strain rate: \( \sigma = \sigma(\varepsilon, \dot{\varepsilon}) \)
- A fitted surface leads then to stress-strain relations for tabulated input
- Neuronal network in LS-OPT works similar
Hill’s functional in MAT181 allows for a proper consideration of Poisson’s ratio ($\nu=0.25$) and yields to a better agreement to the experiment.
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Although thermoplastics do not show a strict transition from elasticity to plasticity, a elasto-(visco)plastic model is (so far) the best choice:

- permanent deformation, implemented for shell elements
- von Mises yield surface still standard for simulation of plastics
- stable simulation; user-friendly input data (e.g. MAT24 in LS-DYNA)
- High sophisticated models (SAMP, MF Polymers, …) available now

In what follows, the validation and verification process (e.g. reverse engineering) is demonstrated for MAT_PIECEWISE_LINEAR_PLASTICITY (MAT_24)
V&V Step 1: Revision of the Test Data; Young’s Modulus

- Test data has to be available as engineering stress vs. engineering strain (Excel / ASCII)
- Visual inspection of the data is necessary first. The goal is to obtain a single sufficiently smooth, i.e. non-oscillatory curve for each strain rate:
  - Eliminate strong oscillating curves
  - Scattering at the same strain rate?
    - If yes: take the average of selected curves at the same strain rate, i.e. eliminate outlayers
    - If no: take the average of all tests at the same strain rate
- Determine average Young’s modulus
V&V Step 2: Conversion, Smoothing and Sampling

- True strain: \( \varepsilon = \ln(1 + \varepsilon_0) \)
- True stress: \( \sigma = \sigma_0 (1 + \varepsilon_0) \)
- This step may be skipped if (local) true stress-strain data is available
- Compute yield curves for each strain rate
- 100 data points are required in the input, thus sampling of the data is necessary:

\[
\varepsilon^1 = 0
\]

\[
\varepsilon^n = \frac{nN}{100}, \quad n = 2, 3, \ldots, 100
\]

\[
\sigma^1 = 0
\]

\[
\sigma^n = \frac{1}{k_e - k_b + 1} \sum_{i=k_b}^{k_e} \sigma^i, \quad n = 2, 3, \ldots, 100
\]

\[
k_e = \min \left( N, \frac{N}{50} (i+1) \right), \quad k_b = \max \left( 1, \frac{N}{50} (i-1) \right)
\]
V&V Step 3: Extrapolation after Necking

- Derive the smoothed curve (that is obtained in step 2) numerically by central difference scheme

\[
\frac{d\sigma}{d\varepsilon} = \frac{\sigma_{n+1} - \sigma_{n-1}}{\varepsilon_{n+1} - \varepsilon_{n-1}}
\]

- Identify the onset of the material instability (necking), i.e. find

\[
\sigma - \frac{d\sigma}{d\varepsilon} = 0 \Rightarrow \varepsilon^*
\]

where \(\varepsilon^*\) is the strain where necking occurs.

If there is an intersection, compute for each strain \(\varepsilon > \varepsilon^*\):

\[
\sigma = \sigma^* e^{(\varepsilon - \varepsilon^*)}
\]

where \(s^* = s(\varepsilon^*)\)

Else Compute the hardening curve:

\[
\sigma_y = \sigma, \quad \varepsilon^p = \varepsilon - \frac{\sigma}{E}
\]
V&V Step 4: Tensile Test Simulation

- Von Mises (piecewise linear) plasticity, linear elastic visco-plastic,
- Generally good representation of tensile responses
V&V Step 4: Tensile Test Simulation (Loop!)

- Compare force-displacement-curve for each strain rate:
  - Correlation must be exact before necking!
  - If correlation is sufficiently accurate after necking, stop
  - If not, go to step 3 and modify the extrapolation (e.g. automatically by optimization software)
Parameter based Material Laws

- Stress-Strain Relation by Schmachtenberg

\[ \sigma = E\varepsilon \frac{1-D_1\varepsilon}{1+D_2\varepsilon} \]

- Example:
  Tensile test

![Graph showing stress-strain relationship for original and Schmachtenberg models, with typical engineering stress in GPa on the y-axis and strain on the x-axis.]
Parameter based Material Laws

- Parameter-identification performed by least square fit

\[ S(E, D_1, D_2) := \sum_{k=1}^{n} \left[ \sigma_k(\varepsilon_k) - E \varepsilon \frac{1-D_1\varepsilon}{1+D_2\varepsilon} \right]^2 \rightarrow \text{MIN} \]

with a gradient method

\[ x^{k+1} = x^k - \alpha \nabla S^k \]

\[ x^k = [E, D_1, D_2]^k, \]

\[ \nabla S^k = \left[ \frac{\partial S}{\partial E}, \frac{\partial S}{\partial D_1}, \frac{\partial S}{\partial D_2} \right]^k \]

\[ \alpha = \text{damping parameter} \]
Parameter based Material Laws

- Strain-Rate Dependency by Johnson Cook
  
  \[ \sigma_y(\dot{\varepsilon}, \varepsilon_p) = \sigma_y(0, \varepsilon_p) \left(1 + \ln \left( \frac{\dot{\varepsilon}}{C} \right) \right)^{\frac{1}{p}} \]
  
  Compute curves for each strain rate
  
  tabulated input in MAT_24

- Cowper Symonds
  
  \[ \sigma_y(\dot{\varepsilon}, \varepsilon_p) = \sigma_y(0, \varepsilon_p) \left[1 + \left( \frac{\dot{\varepsilon}}{C} \right)^{\frac{1}{p}} \right] \]
  
  Parameters \( C, p \) can be used directly in the MAT_24 card
Material Models of Polymers for Crash Simulation

And now ...

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