

15. Deutsches LS-DYNA Forum

Updated Review of Solid Element Formulations in LS-DYNA Properties, Limits, Advantages, Disadvantages

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Motivation

Solid elements are three-dimensional finite elements that can model solid bodies and structures without any a priori geometric simplification.

- No geometric, constitutive and loading assumptions required.
- Boundary conditions treated more realistically. (compared to shells or beams).
- FE mesh visually looks like the physical system.

but...

- Higher effort: mesh preparation, CPU time, post-processing, …
- Expensive mesh refinement: Curse of dimensionality.
- Often poor performance for thin-walled structures (locking problems).





Applications

- Foam Structures
- Rubber components
- Cast iron parts
- Solid barriers
- Plastic parts
- Bulk forming
- Thick metal sheets
- Elastic tools

....

Impact analysis





Overview

LS-DYNA User's manual: *SECTION_SOLID, parameter ELFORM

- EQ.-2 Fully integrated S/R solid intended for elements with poor aspect ratio, accurate formulation
- EQ.-1 Fully integrated S/R solid intended for elements with poor aspect ratio, efficient formulation
- EQ.1 Constant stress solid element: default element type
- EQ.2 Fully integrated S/R solid.
- EQ.3 Fully integrated quadratic 8-node element with nodal rotations



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- EQ.-2 Fully integrated S/R solid intended for elements with poor aspect ratio, accurate formulation
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Standard hexahedral elements

EQ.1 Constant stress solid element: default element type

EQ.2 Fully integrated S/R solid.

EQ.3 Fully integrated quadratic 8-node element with nodal rotations

EQ.4	S/R quadratic tetrahedron element with nodal rotations
EQ.10	1 point tetrahedron
EQ.13	1 point nodal pressure tetrahedron
EQ.15	2 point pentahedron element
EQ.16	4 or 5 point 10-noded tetrahedron
EQ.17	10-noded composite tetrahedron
EQ.23	20-node solid formulation
EQ.24	27-noded, fully integrated S/R quadratic solid element
EQ.115	1 point pentahedron element with hourglass control

Standard hexahedra elements in LS-DYNA

ELFORM = 1



- underintegrated constant stress
- efficient and accurate
- even works for severe deformations
- needs hourglass stabilization: choice of hourglass formulation and values remains an issue

ELFORM = 2



- selective reduced integrated brick element (volumetric locking alleviated)
- slower than ELFORM=1
- more unstable in large deformation applications
- no hourglass stabilization needed
- too stiff in many situations, especially for poor aspect ratios (shear locking)



Hourglass control for ELFORM=1

*HOURGLASS: IHQ = 1...5

- viscous form (1,2,3) for higher velocities
- stiffness form (4,5) for lower velocities
- exact volume integration recommended (3,5)

*HOURGLASS: IHQ = 6

- the QBI (Quintessential Bending Incompressible) hourglass control by Belytschko and Bindeman
- hourglass stiffness uses elastic constants
- recommended in most situations
- sometimes modified QM makes sense (watch hourglass energy)

*HOURGLASS: IHQ = 7/9

- similar to type 6, but less experience
- type 7 uses total deformation instead of updated
- type 9 should provide more accurate results for distorted meshes





Property of ELFORM=2

Shear locking

- Pure bending modes trigger spurious shear energy
- Getting worse for poor aspect ratios

 $\varepsilon_{xx} = 2\xi_y/l_x, \ \varepsilon_{yy} = 0, \ \gamma_{xy} = \xi_x/l_y$

Alleviation of shear locking

- 1. Underintegration \rightarrow ELFORM = 1
- 2. Modified strain formulations \rightarrow modified Jacobian matrix



Bending moment

 $\gamma \neq 0$

V

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Solid element formulations -1 and -2

Thomas Borrvall: "A heuristic attempt to reduce transverse shear locking in fully integrated hexahedra with poor aspect ratio", Salzburg 2009

Original Jacobian matrix

$$J_{ij}^{\text{orig}} = \frac{\partial x_i}{\partial \xi_j} = x_{Ii} \frac{1}{8} \left(\xi_j^I + \xi_{jk}^I \xi_k + \xi_{jl}^I \xi_l + \xi_{123}^I \xi_k \xi_l \right)$$

Modification of the Jacobian matrix

Reduction of spurious stiffness without affecting the true physical behavior of the element

$$J_{ij}^{\text{mod}} = x_{Ii} \frac{1}{8} \left(\xi_j^{I} + \xi_{jk}^{I} \xi_k \kappa_{jk} + \xi_{jl}^{I} \xi_l \kappa_{jl} + \xi_{123}^{I} \xi_k \kappa_{jk} \xi_l \kappa_{jl} \right)$$

aspect ratios between dimensions computed efficiently

- computed once at time zero
- with edge lengths through element center



Improved hexahedral elements

Identical with ELFORM=2 but accounted for **poor aspect ratio** in order to reduce shear locking



ELFORM = -1

- efficient formulation
- sometimes hourglass tendencies

ELFORM = -2

- **accurate** formulation
- higher computational cost than type -1

CPU cost compared to ELFORM=2 for explicit analysis ≈ 1.2 (ELFORM=-1) ≈ 2.0 to 3.5 (ELFORM=-2)

... but implicit ELFORM -2 is a good choice!



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Quadratic hexahedron element formulations (use R10)

Good bending behaviour



- Serendipity formulation not fully quadratic
- Faster than ELFORM 24



- Fully quadratic displacement field
- 21 degrees of freedom more than ELFORM 23
- S/R integration

Compared to 8-node solid hexahedral "coarse meshes" often sufficient for convergence



Quadratic hexahedron element formulations (use R10) Automatic node generation similar to *ELEMENT_SOLID_TET4TOTET10



- Boundary conditions partly translated
- Size of time step getting smaller



Quadratic hexahedron element formulations (use R10)

Define nodes a priori with mesh generator / LSPP





Quadratic hexahedron element formulations (use R10)

Define nodes a priori with mesh generator / LSPP

Using *ELEMENT_SOLID_H20/_H27 all mid-side nodes may be directly accessed e. g. for single point constraints

*ELI	EMENT S	OLID H27						
\$#	eid	pid						
	1	1						
\$#	nl	n2	n3	n4	n5	n6	n7	n8
	1	2	3	4	5	6	7	8
\$#	n11	n12	n13	n14	n15	n16	n17	n18
	11	12	13	14	15	16	17	18
\$#	n21	n22	n23	n24	n25	n26	n27	
	21	22	23	24	25	26	27	







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Quadratic element with rotation

EQ.3 Fully integrated quadratic 8-node element with nodal rotations

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Quadratic solid element with nodal rotations

ELFORM = 3

- Quadratic 8 node hexahedron with nodal rotations, i.e. 6 DOF per node
- Full integration (12 point)
- Well suited for connections to shells
- Good accuracy for small strains
- Tendency to volumetric locking possible remedy: incompatible modes

Pawlak, TP and Yunus, SM: Solid elements with rotational degrees of freedom: Part 1 – Hexahedron elements, IJNME 1991



8 nodes with 48 DOFs

20 nodes with 60 DOFs

Teng, H: Solid elements with Rotational Degree of Freedom for Grand Rotation Problems in LS-DYNA, 11th International LS-DYNA Users Conference, 2010



Examples part 1





Implicit elastic bending

- clamped plate of dimensions 10x5x1 mm³
- subjected to 1 Nm torque at the free end
- E = 210 GPa
- analytical solution for end tip deflection: 0.57143 mm
- convergence study with aspect ratio 5:1 kept constant



End tip deflection, different mesh c	discretizations and element types,	error in parenthesis.
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Mesh	ELFORM 2	ELFORM -2	ELFORM -1	ELFORM 3	ELFORM23	ELFORM 24
2×1×1	0.0564(90.1%)	0.6711(17.4%)	0.6751(18.1%)	0.4001(30.0%)	0.5264(7.8%)	0.5525(3.3%)
4 x 2 x 2	0.1699(70.3%)	0.5466(4.3%)	0.5522(3.4%)	0.4596(19.6%)	0.5456(4.5%)	0.5534((3.1%)
8×4×4	0.3469(39.3%)	0.5472(4.2%)	0.5500(3.8%)	0.5237(8.4%)	0.5517(3.5%)	0.5541(3.0%)
16×8×8	0.4820(15.7%)	0.5516(3.5%)	0.5527(3.3%)	0.5557(2.8%)	0.5537(3.1%)	0.5543(3.0%)
32×16×16	0.5340(6.6%)	0.5535(3.1%)	0.5540(3.1%)	0.5552(2.8%)	0.5543(3.0%)	0.5545(3.0%)



Plastic bending

- Explicit plastic 3 point bending (prescribed motion)
- Plate of dimensions 300×60×5 mm³
- *MAT_024 (Aluminum)











Circular tube compression

- *MAT_024 (Aluminium)
- Different mesh sizes: convergence study





















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MORE

Tube crash





Tube crash





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Linear tetrahedral elements in LS-DYNA

ELFORM = 10



- 1-point constant stress
- Volumetric locking stiff behavior
- Only applicable for foams with v = 0 (not recommended in general)
- Often used for transitions in meshes *CONTROL_SOLID, ESORT=1

ELFORM = 13



- 1-point constant stress with nodal pressure averaging to alleviate volumetric locking
- Better performance than ELFORM=10 if Poisson's ratio v > 0 (metals, rubber, ...)
- Supported materials for explicit
 *MAT_001, 003, 006, 007, 015, 024, 027, 077, 081, 082, 091, 092, 098, 103, 106, 120, 123, 124, 128, 129, 181, 183, 187, 224, 225, 244
- Implicit: all materials supported



Linear tetrahedra elements in LS-DYNA Theoretical background

Bonet J, Burton, AJ: A simple average nodal pressure tetrahedral element for incompressible dynamic explicit applications. Comm. Num. Meth. Engrg. 14: 437-449, 1998

"(...) the element **prevents volumetric locking** by defining nodal volumes and evaluating average nodal pressures in terms of these volumes (...)"

"(...) it can be used in explicit dynamic applications involving (nearly) **incompressible material behavior** (e.g. rubber, ductile elastoplastic metals) (...)"



Speed penalty of max. 25% compared to TET#10



- = TET 10 + averaging nodal pressures
- = TET 10 volumetric locking



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Higher o	rder tetrahedral elements
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Higher order tetrahedral elements

ELFORM = 16



- 4(5) point 10-noded tetrahedron
- Good accuracy for moderate strains
- High cpu cost
- Observe the node numbering
- Use *CONTACT_AUTOMATIC_... with PID
- Easy conversion of 4-noded tets via
 *ELEMENT_SOLID_TET4TOTET10
- Midside nodes: *CONTROL_OUTPUT, TET10=1

ELFORM = 17

- 4(5) point 10-noded "composite" tetrahedron (12 linear sub-tetrahedrons)
- Properties similar to type 16
- Correct external force distribution

Automatic node generation	
*ELEMENT_SOLID_TET4TOTET10	
Similar to *ELEMENT_SOLID_H8TOH20/_H8TOH27	



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Quadratic tetrahedron with nodal rotations

ELFORM = 4

- Quadratic 4 node tetrahedron with nodal rotations, i.e. 6 DOF per node
- Derived from 10 node tetrahedron
- S/R integration (5-point)
- Well suited for connections to shells
- Good accuracy for small strains
- Tendency to volumetric locking

Quasi-quadratic

Teng, H: Solid elements with Rotational Degree of Freedom for Grand Rotation Problems in LS-DYNA, 11th International LS-DYNA Users Conference, 2010

Pawlak, TP and Yunus, SM: Solid elements with rotational degrees of freedom: Part 2 – Tetrahedron elements, IJNME 1991



Examples part 2





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Plastic bending

- Explicit plastic 3 point bending (prescribed motion)
- plate of dimensions $300 \times 60 \times 5 \text{ mm}^3$
- *MAT_024 (aluminum)



























Rubber block compression

Sphere pushed into rubber block









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Structural component



Load-displacement curve







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MORE

Structural component



2 – 10 mm





1 – 5 mm



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Structural component





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Pentahedra elements in LS-DYNA

ELFORM = 15



- 2-point selective reduced integration
- needs hourglass stabilization for twist mode (recent improvement → next official versions)
- often used as transition element (ESORT=1)

ELFORM = 115



- 1-point reduced integration
- needs hourglass stabilization (analogue to hexahedron element type 1 with Flanagan-Belytschko hourglass formulation)

should in general be avoided – better: pure hexahedral or tetrahedral meshes



Time step control

Critical time step

$$\Delta t_e = \frac{L_e}{Q + (Q^2 + c^2)^{1/2}} \approx \frac{L_e}{c}$$

 L_e = main information from element formulation

Adiabatic sound speed

$$c = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}} = \sqrt{\frac{K + \frac{4}{3}G}{\rho}}$$

Characteristic element length

ELFORM = 1, 2, 3, -1, -2:
$$L_e = V/A_{max}$$

ELFORM = 4: $L_e = 0.85 h_{min}$
ELFORM = 10 / 13: $L_e = h_{min}$
ELFORM = 16: $L_e = 0.3889 h_{min}$
ELFORM = 17: $L_e = V/A_{max}$
ELFORM = 15: $L_e = 1/\sqrt{B_{ij}B_{ij}}$



Time step size for same volume 1.00 1.42 0.68 0.65 0.56 0.27 0.35 1.2,3,4 1.01 0.68 0.65 0.56 0.27 0.35 23 24



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Conclusions and remarks



- Use hexahedron elements if possible (regular solid bodies)
 - ELFORM = 1 with IHQ = 6 or ELFORM = 2, 3
 - ELFORM = -1 or -2 for "flat" hexas
- Quadratic ELFORM 23, 24 show good coarse mesh accuracy
- But for large strains linear elements in general more robust





- Pentahedrons 15/115 should be avoided or only be used as transition elements, pure tetrahedral mesh is better choice
- General remarks
 In implicit analysis: costly element formulations may be used not as significant for speed as in explicit analysis
 Always set ESORT = 1 on *CONTROL_SOLID



Conclusions and remarks



- For complex solid structures, use tet type 4, 13, 16, or 17
 - ELFORM = 16/17 are the most accurate tets, but not suited for large strains
 - ELFORM = 13 needs finer mesh, well suited even for large strains (check if your material is supported)
- For metals or plastics (moderate strains), use tet type 4, 13, 16, or 17
- For rubber materials (incompressible, large strains) use tet type 13
- For bulk forming problems (large strains!), use ELFORM = 13 and r-adaptivity





Conclusions and remarks

*SECTION_SOLID, parameter ELFORM

- EQ.25 21-noded quadratic pentahedron
- EQ.26 15-noded quadratic tetrahedron
- EQ.27 20-noded cubic tetrahedron
- EQ.28 40-noded cubic pentahedron
- EQ.29 64-noded cubic hexahedron

Further quadratic and cubic element types available in more recent versions of LS-DYNA

www.dynalook.com

Teng, H: "Recent Advances on Higher-Order 27-node Hexahedral Element in LS-DYNA", 14th International LS-DYNA Users Conference 2016.

Borrvall, T, Benson, D, Teng, H: "An Implicit Study of High Order Elements in LS-DYNA", 11th European LS-DYNA Conference 2017.