

# FEA Information

## WORLDWIDE NEWS



JUNE 2004

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**Letter To The Engineering Community  
Trent Eggleston & Marsha Victory**

**June 24, 2004**

**This June issue marks another milestone in attaining the goals of FEA Information Inc.'s monthly newsletter and websites.**

**We are expanding our format in order to better serve the engineering community. Therefore, we've added new sections.**

**During the ensuing months you will notice continuing expansion in the scope of the News. This month we've added:**

- **Directories**
- **Community Announcements**
- **NewsWire**

**Feel free to contact us in order to contribute an article, or to become an active participant for the engineering community.**

**Sincerely,**

***Trent Eggleston & Marsha Victory***



## Using LS-DYNA for Heat Transfer & Thermal-Stress Problems

A short course taught by: Art Shapiro, Ph.D.

Class Location: Livermore Software Technology Corporation  
7374 Las Positas Road - Livermore, CA 94550

Contact: Denette Trowbridge - 925-245-4505 phone  
denette@lstc.com

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### Objective of the course

This course objective is to provide an understanding of computational finite element heat transfer. Presentations 1-8 focus on the various heat transfer modeling issues one must understand in using LS-DYNA. This is followed by an introduction to thermal-stress and thermal-fluid problems. Workshop problems are used to illustrate the points made in the lectures.

Lectures begin daily at 9:00 a.m. and run until 5:30 p.m. The classroom machines are PCs running on the Linux operating system.

### Course Contents

1. Introduction – Learn to create a KEYWORD input file to solve for the thermal expansion of an aluminum block.
2. Mathematical Theory – brief, but can't be avoided.
3. Time Step Control – Learn how to select a time step size, use the variable time step option, and understand the difference between fully implicit and Crank Nicolson time integration methods.
4. Boundary Conditions – Learn how to define temperature, flux, convection, and radiation boundary conditions. Learn how to hand calculate a convection heat transfer coefficient.
5. Nonlinear Problems – Learn nonlinear heat transfer by solving a solid-liquid phase change problem.
6. Equation Solvers – Learn the advantages and disadvantages between the Gauss direct solvers & conjugate gradient iterative solvers in LS-DYNA.
7. Thermal Contact – Learn thermal contact modeling issues by solving a sheet metal forming problem with thin and thick shells.
8. Miscellaneous – Learn special applications including powders, welding, induction heating, and thermostat control.
9. Thermal-stress coupling – An introduction to coupled thermal stress modeling with upsetting, forging, extrusion, and sheet metal forming applications.
10. ALE coupled thermal mechanics – How to use ALE for very large deformation thermal stress problems.
11. Thermal-fluid-coupling - An introduction to coupled thermal fluid modeling with casting applications.

## **EXECUTIVE SUMMARY**

### **Challenge:**

To calculate numerous physical values and create gear tooth models for UH-60 helicopters in U.S. Navy aircraft fleet.

### **Solution:**

Implement ANSYS software to perform to convert gear tooth models into a finite element modeling and analysis (FEM/FEA) mesh that can be analyzed.

### **Benefits:**

Increased mission reliability, better scheduled maintenance to reduce aircraft downtime, and dramatically decreased life cycle costs.

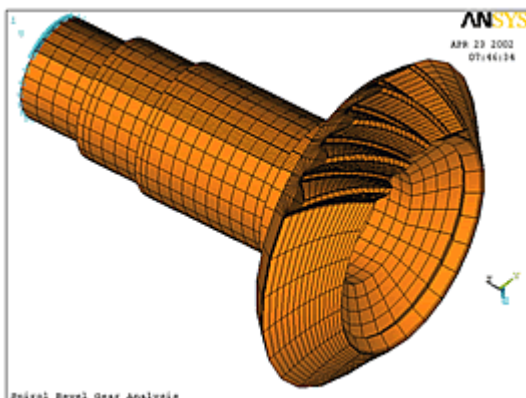
Identified sensitivities and uncertainties in the effects of material properties and manufacturing defects on component capacity.

Provided invaluable calibration of a prognostic model at various times in the life of a component so that it can be evaluated both in terms of long-term capability prediction (asset management) and more near-term damage minimization (fault accommodation).

### **Introduction:**

For 20 years, the U.S. has asked its military to do more with less. Even as resources are trimmed, the armed forces are asked to take on new missions in faraway places. All too often, this is done with equipment that is far beyond the service life for which it was designed. Despite their increasing age, aircraft remain in service because replacements are not being acquired.

As necessity is the mother of invention, so budget constraints and readiness pressures have led to analyzing problems and finding new solutions. Among the most scientific and ingenious is a program focused on gears from UH-60 helicopter transmissions in U.S. Navy aircraft.



**Initially meshed ANSYS model of the spiral bevel pinion from the intermediate gearbox of a UH-60 helicopter. The steel is AISI 9310**

The Naval Air Systems Command has done extensive predictive and diagnostic work at its Helicopter Transmission Test Facility (HTTF) at Patuxent River. Funding is from the Defense Advanced Research Projects Agency (DARPA). The program also reaches down to consultants and engineering analysts in Upstate New York. Impact Technologies LLC, in Rochester, and a subcontractor in nearby Ithaca, Fracture Analysis Consultants Inc. (FAC), has undertaken analytical work.

Holding all this together is ANSYS Mechanical design and optimization software, which was used to calculate numerous physical values, to create the gear tooth models from proprietary gear-design geometry, and to convert that model into a finite element modeling and analysis (FEM/FEA) mesh for analysis.

### **Challenge:**

Prognostics builds on 40-plus years of aircraft diagnostics and trouble-shooting. Diagnostic systems provide increased safety, but unfortunately, “they can require pulling components early while they still have a remaining useful life,” said William E. Hardman of the Naval Air Warfare Center Aircraft Division's Propulsion & Power Department, Patuxent River.

The goal is to create on-board systems to help commanders, maintenance crew chiefs and pilots make “go/no go” decisions in situations from routine maintenance to battlefields and rescues. Bringing prognosis capability on-board, NavAir believes, eliminates the potential for lost data, data dropout, and incorrectly processed data at the ground station.

Hardman, DARPA, NavAir and Patuxent River experts are building on their understanding of the relationships between existing diagnostic capability, damage evolution models, and rules and tools for prognosis of power drive train systems in combat aircraft. These are being integrated with the sciences of fracture mechanics (and fault progression), vibration-based mechanical diagnostics, component failure prediction, statistics, and material science.

In the program's early stages, NavAir experts evaluated statistical risk-reduction methods in terms of available and advanced mechanical diagnostics technologies. This was done in a series of seeded and propagation-fault tests. (In a “seeded” test, a crack or fault is intentionally induced rather than waiting for it to occur on its own.).

At the end of its work for NavAir, Impact concluded that useful and reliable prognostic models for helicopter gears can be built that will predict remaining life as a function of speed and load.

### **Solution:**

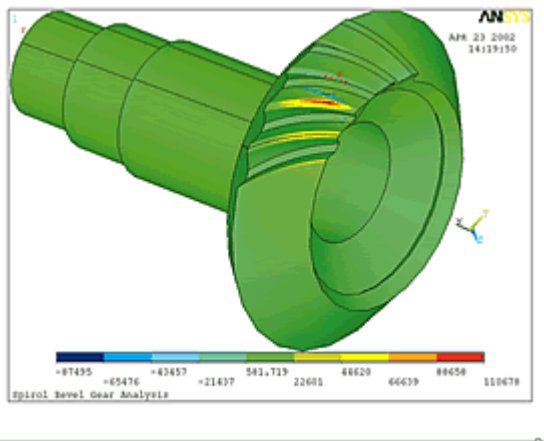
Geometry and loading for three gear teeth were obtained in the form of finite elements written in ANSYS format from a proprietary gear-generation program, explained Lamirand, an Impact analyst who worked closely with Greg Kacprzyński, project manager for prognostics work at Impact, and Avinash Sarlashkar of Impact, who with Brad Lamirand, did the ANSYS work.

Using ANSYS, the model was expanded into a full 3-D model. “We did multiple static analyses at each tooth load increment,” said Lamirand. “These were post-processed in ANSYS to establish the static stresses over engagement of the three teeth. The crack propagation and trajectory model was developed working with the fully meshed ANSYS model.

The initial ANSYS model was meshed with about 31,000 ten-node Solid92 tetrahedral elements. The model was started with proprietary formatted gear data, plus associated load definitions for the pinion's load history when teeth are completely engaged with a mating gear. The fracture mechanics model had 920,000 degrees of freedom and used 13 crack fronts, also known as load steps.

“Each load step was subjected to 18 load cases simulating instantaneous pinion loads,” Kacprzynski noted. “These corresponded to 18 discrete angular positions during the load-unload cycle for the pinion. As the crack grew, more elements were required to resolve the geometric details of the crack surface. By the 12th load step, the model reached nearly 1.4 million DOFs.”

The modeling effort was not to figure out what happens with a cracked gear tooth, but to see how high a level of confidence could be built into the prognostic model.



ANSYS model of the spiral bevel pinion gear from the intermediate gearbox of a UH-60 helicopter, showing areas of gear teeth that see maximum stress when engaged.

Kacprzynski's results showed that the mean predicted time to crack initiation was 11 hours while the actual time was 15 hours. In a second test, the mean prediction was 2.38 hours and the actual time was approximately 2.5 hours. This put the prognostics in the 98th and 63rd percentiles for accuracy and yielded (statistical) standard deviations of 1.64 and 0.36 hours, respectively.

The ANSYS calculations required tracking a moving load on the gear tooth surface with a continuously changing point of contact,” Sarlashkar explained. “There is a changing magnitude of load, which reaches its maximum as the tooth fully engages, then decreasing as the tooth disengages. “This was handled with ANSYS multi-load analysis as a function of the tooth engagement,” he added. “This was a linear-elastic problem in an isotropic and homogeneous material using the preconditioned conjugate gradient solver.”

### Benefits:

The ANSYS work by Impact was aimed at “developing validated models for predicting asset readiness,” Kacprzynski wrote. In his IEEE paper, he pointed out that NavAir results are used to:

Provide realistic indications of state awareness, that is, health indicators such vibration features, and non-destructive evaluations related to actual material damage levels.

**Identify sensitivities and uncertainties in the effects of material properties and manufacturing defects on component capacity.**

**Provide invaluable calibration of a prognostic model at various times in the life of a component so that it can be evaluated both in terms of long-term capability prediction (asset management) and more near-term damage minimization (fault accommodation).**

**The key to a successful prognostic model for pinions is to link state awareness to material properties. “The software module,” he explained, “integrates advanced stochastic failure mode modeling, failure progression information from vibration features, and run-to-failure experience bases to enable IGB pinion gear failure predictions in the H-60 critical drive train.**

**“The failure rate prediction strategies are implemented within a probabilistic framework to directly identify confidence bounds associated with IGB pinion failure progression,” he added. “The results of seeded fault, run-to-failure tests on the IGB pinion gear are being compared to prognostic module predictions.”**

**Impact's results were verified in additional Patuxent tests. A pinion gear was driven to failure three times and Impact's prognoses closely matched what actually occurred in the test stand. “Mission reliability could be greatly increased, maintenance could be better scheduled to reduce aircraft downtime, and a dramatic decrease in life cycle costs could be realized,” Hardman said.**

**“What the ANSYS models provided us,” concluded Kacprzyński, “is the foundation upon which high-fidelity prognostic models for helicopter gears can be built that will predict remaining life as a function of speed and load.”**



**The NEC Express5800/1000 series**  
**© Copyright NEC**

The NEC Express5800/1000 series includes servers which can be configured with up to 32 of Intel's new 64-bit processors, "Intel® Itanium® 2 Processor." NEC is a Microsoft® Windows Server™ 2003 OEM and the Express5800/1000 Series is available with Windows Server™ 2003 Enterprise Edition (model 1080Xd only), or Windows Server™ 2003 Datacenter Edition (models 1160Xd and 1320Xd).

**Main features of the Express5800/1000 series include**

**High-speed processing and high scalability**

The NEC Express5800/1320Xd posts a record TPC benchmark! High-speed processing is realized through combining Intel's new CPU and NEC's technology -- such as crossbar switch technology and high-speed memory access technology -- which were cultivated through years of developing supercomputers and main frames. In addition to the fact that it can be configured with up to 32 CPUs, the new series employs PCI-X for I/O bus to connect peripheral and network devices. It can also be configured with up to 112 slots, demonstrating the high scalability ideal for large-scale configurations. The series has achieved industry-record performance on LINPACK HPC of 101.77GFLOPS for a 32 CPU server which employs general purpose MPU.

**Partitioning function for flexible system operation**

The Express5800/1000 series may be partitioned at the hardware level, enabling each subsystem to operate as an independent computer system. It is possible to divide the 32 CPU system into up to eight subsystems, each of which has four CPUs, and let them each operate separately. In this way it's possible to allocate processor capacity according to workload status, resulting in flexible system operation and guaranteed security.

**High reliability and availability**

Many hardware components (clock, power, fan, service processor, etc.) allow redundancy, and the CPU memory board and I/O card hardware are hot-swap compliant. Easy system recovery is possible even at the time of failure.

**Less is More...until it's Less**  
**Virtual Consolidation: Connecting Distributed Teams.**  
© Copyright SGI  
<http://www.sgi.com/features/2003/oct/less/>

The principal challenge of every global company IT manager is installing and maintaining an IT infrastructure that has the ability to adapt quickly to a changing business and takes full advantage of price and performance. At a time when so many industries are becoming distributed across the globe with ever-smaller staffs, that task becomes all the more challenging. SGI provides the high-performance computing, visualization and storage tools to keep global companies functioning smoothly and profitably.

The war cry of companies governed by Moore's Law is "Less is More." This is a well-lit path down which consumers of high technology systems encounter surprisingly improved capabilities at ever-declining costs. But these improvements are principally incremental versions of the same theme, with rare significant changes-with the notable exception of systems provided by Silicon Graphics for high performance, scalable, extensible, adaptable and distributed storage, computing and graphics.



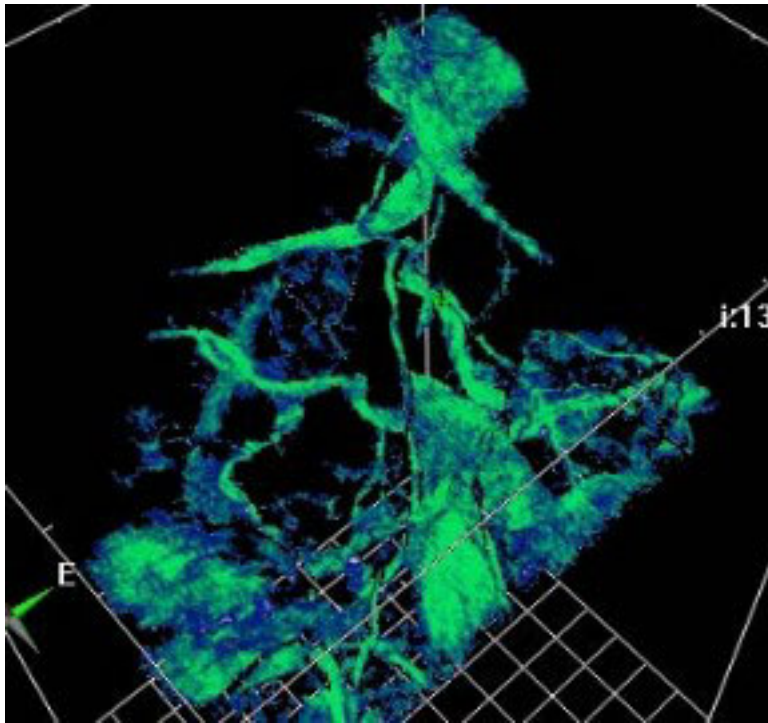
**VoxelVision showcases GigaVIZ on SGI Altix at the June convention of the European Association of Geoscientists and Engineers in Stanvanger, Norway**

SGI's new Linux® OS-based Altix® servers provide unprecedented scalability and performance. Fed by the SGI® InfiniteStorage products, CXFS™ with shared filesystems delivers unmatched and unmatchable system performance. Completing the triad, the new ATI-enabled Silicon Graphics® Onyx4™ visualization system is the scalable, standards-based graphics solution that is revolutionizing not only SGI® Reality Center® and team room environments, but also power user and distributed and collaborative graphics throughout the technical computing enterprise.

Relying only on Moore's law will dead-end if it doesn't spawn fundamental change in-rather than a mere acceleration of-the way we work. In Information Systems, as in sport cars, piecemeal assembly of components cannot create an optimized experience. High-performance systems such as the SGI® Altix® 3000 family and Onyx4 are not accidents but rather the intentional, balanced combination of optimized systems, joined with care and focused to fit the purpose.

The energy industry, majors and independents alike, seeks work processes to support a challenging technical demographic, an increasingly distributed workforce, a step-function explosion of new data and expectations of ever more competitive financial performance. While existing systems are fast and

affordable, they falter in the scalability and real-time flexibility that are the fundamentals of the next generation of information systems



**Complex volumetric seismic attributes are isolated, imaged and visualized in GigaVIZ by VoxelVision powered by Altix from SGI.**

The business thus presents a challenge: seamlessly and efficiently support a decentralized workforce while retaining the cost benefits of a centralized company. These contrasting themes must be married to support advanced reservoir management and field optimization, along with outsource-heavy data management, computing and analysis. Exploration and production will benefit from this union, streamlining data management and access, interlocking the experience of global workers, saturating investment and operational decisions with both data and knowledge and delivering breakthroughs in cost containment and financial performance. Even more importantly, this will take fullest advantage of today's workforce juggernaut-the experience-rich, independent consultant

In tandem with the hardware architectural revolution comes new and innovative software delivering breakthrough capabilities in fresh ways. A new product has come to market, GIGAviz™ from VoxelVision, that addresses the collaboration needs of the data-hungry energy industry through the pioneering Altix system for visualization. VoxelVision's advanced 3D-visualization system powered by the SGI Altix server is set to revolutionize quality control of seismic data processing in the oil and gas industry.

The award-winning SGI Altix 3000 server-the industry's most scalable processing technology based on the 64-bit Intel® Itanium® 2 processor, high bandwidth NUMAflex™ system architecture and the industry standard Linux operating system-runs GIGAviz to provide 3D visualization and interactive interpretation of virtually unlimited data volumes.

SGI, with our partners' serving such industries as oil and gas are creating a new infrastructure termed Virtual Consolidation. Unlike many of today's systems, which ask businesses to adapt to the technology, Virtual Consolidation adapts to business needs. These systems must face and convincingly conquer the reality that the rate of change in technology is measured in years, while the rate of change

of business can be as short as months. Technologies that may be perfect for today but not adaptable for tomorrow will transform seemingly overnight into severe encumbrances to business growth and performance.



**SGI Altix driving GigaVIZ from VoxelVision to a full booth at the June convention of the European Association of Geoscientists and Engineers in Stavanger, Norway**

Businesses held by the sharp talons of inflexible systems will have but two choices: wait for business needs to return to the capabilities of their information systems, or replace their systems with something new and more appropriate. While neither choice offers near-term relief, only the second offers a path forward -and, at that, only if the choice is the flexibility, scalability and adaptability SGI delivers. Otherwise, the new system risks coming on line just in time to vie for "most obsolete" against the very one it just replaced.

Virtual Consolidation is a simple concept. Data, storage, applications, computational servers, graphical servers, people, processes and services are distributed according to distribution of the business. As assets mature, are acquired, are sold or are re-invigorated, or as the finances of the business change through reduction in operating costs, incentives from governments, increase in low-cost capital or increase in commodity pricing, the information systems that support them adapt in real time. All of this exists, adapts, responds and performs, yet the view to the enterprise looks like it's contained in a single instance. This means that any user sees a single file system (make no copies, make no versions), can access all computational servers with high-speed data access, connect to real-time high performance visualization, participate in realistic collaboration and be provided instant service regardless of his or her location or desktop device.

Concepts do not solve business problems; solutions do. SGI serves a cross-section of global industries, sowing the seeds of Virtual Consolidation. The catalyst to harvesting this bounty will be insightful customers with clear visions of their own uncertain futures.

It is uncertainty that drives this revolution. It is uncertainty that must be tamed. It is uncertainty that will be an ally rather than a foe. But like in that sports car, the right components assembled in just the right way will allow an ace driver to navigate unexpected sharp turns at full velocity, maintain the center of the lane with no extra cost and remain ahead of their competitors. Likewise, the next information systems for global companies must adapt to both the unexpected and planned tactical turns. The Virtual Company is the vehicle, Virtual Consolidation the engine. All available from SGI today.

*Hardware  
&  
Computing and Communication Products  
(Listed in Alphabetical Order)*



[www.amd.com](http://www.amd.com)



[www.fujitsu.com](http://www.fujitsu.com)



[www.hp.com](http://www.hp.com)



[www-1.ibm.com/servers/deepcomputing](http://www-1.ibm.com/servers/deepcomputing)



[www.intel.com](http://www.intel.com)



[www.nec.com](http://www.nec.com)



[www.sgi.com](http://www.sgi.com)

# Software Distributors

*Alphabetical order by Country*

<b>Australia</b>	<b>Leading Engineering Analysis Providers</b> <b><a href="http://www.leapaust.au">www.leapaust.au</a></b>
<b>Canada</b>	<b>Metal Forming Analysis Corporation</b> <b><a href="http://www.mfac.com">www.mfac.com</a></b>
<b>China</b>	<b>ANSYS China</b> <b><a href="http://www.ansys.cn">www.ansys.cn</a></b>
<b>China</b>	<b>MSC. Software – China</b> <b><a href="http://www.mssoftware.com.cn">www.mssoftware.com.cn</a></b>
<b>Germany</b>	<b>CAD-FEM</b> <b><a href="http://www.cadfem.de">www.cadfem.de</a></b>
<b>Germany</b>	<b>DynaMore</b> <b><a href="http://www.dynamore.de">www.dynamore.de</a></b>
<b>India</b>	<b>GissETA</b> <b><a href="http://www.gisseta.com">www.gisseta.com</a></b>
<b>India</b>	<b>Altair Engineering India</b> <b><a href="http://www.altair.com">www.altair.com</a></b>
<b>Italy</b>	<b>Altair Engineering Italy</b> <b><a href="http://www.altairtorino.it">www.altairtorino.it</a></b>
<b>Italy</b>	<b>Numerica SRL</b> <b><a href="http://www.numerica-srl.it">www.numerica-srl.it</a></b>
<b>Japan</b>	<b>Fujitsu Limited</b> <b><a href="http://www.fujitsu.com">www.fujitsu.com</a></b>
<b>Japan</b>	<b>The Japan Research Institute</b> <b><a href="http://www.jri.co.jp">www.jri.co.jp</a></b>
<b>Korea</b>	<b>Korean Simulation Technologies</b> <b><a href="http://www.kostech.co.kr">www.kostech.co.kr</a></b>
<b>Korea</b>	<b>Theme Engineering</b> <b><a href="http://www.lsdyna.co.kr">www.lsdyna.co.kr</a></b>

# *Software Distributors*

*Alphabetical order by Country*

<b>Russia</b>	<b>State Unitary Enterprise <a href="http://www.ls-dynarussia.com">www.ls-dynarussia.com</a></b>
<b>Sweden</b>	<b>Engineering Research AB <a href="http://www.erab.se">www.erab.se</a></b>
<b>Taiwan</b>	<b>Flotrend <a href="http://www.flotrend.com.tw">www.flotrend.com.tw</a></b>
<b>USA</b>	<b>Altair Western Region <a href="http://www.altair.com">www.altair.com</a></b>
<b>USA</b>	<b>Engineering Technology Associates <a href="http://www.eta.com">www.eta.com</a></b>
<b>USA</b>	<b>Dynamax <a href="http://www.dynamax-inc.com">www.dynamax-inc.com</a></b>
<b>USA</b>	<b>Livermore Software Technology Corp. <a href="http://www.lstc.com">www.lstc.com</a></b>
<b>USA</b>	<b>ANSYS Inc. <a href="http://www.ansys.com">www.ansys.com</a></b>
<b>UK</b>	<b>Oasys, LTC <a href="http://www.arup.com/dyna/">www.arup.com/dyna/</a></b>

## *Consulting Services*

### **Alphabetical order by Country**

<b>Australia</b> Manly, NSW <a href="http://www.leapaust.com.au">www.leapaust.com.au</a>	<b>Leading Engineering Analysis Providers</b>  <b>Greg Horner</b> <a href="mailto:info@leapaust.com.au">info@leapaust.com.au</a> <b>02 8966 7888</b>
<b>Canada</b> Kingston, Ontario <a href="http://www.mfac.com">www.mfac.com</a>	<b>Metal Forming Analysis Corporation</b>  <b>Chris Galbraith</b> <a href="mailto:galb@mfac.com">galb@mfac.com</a> <b>(613) 547-5395</b>
<b>India</b> Bangalore <a href="http://www.altair.com">www.altair.com</a>	<b>Altair Engineering India</b>  <b>Nelson Dias</b> <a href="mailto:info-in@altair.com">info-in@altair.com</a> <b>91 (0)80 2658-8540</b>
<b>Italy</b> Torino <a href="http://www.altairtorino.it">www.altairtorino.it</a>	<b>Altair Engineering Italy</b>  <a href="mailto:sales@altairtorino.it">sales@altairtorino.it</a>
<b>Italy</b> Firenze <a href="http://www.numerica-srl.it">www.numerica-srl.it</a>	<b>Numerica SRL</b>  <a href="mailto:info@numerica-srl.it">info@numerica-srl.it</a>  <b>39 055 432010</b>



*Consulting Services*

**Alphabetical order by Country**

<p><b>UK</b> Solihull, West Midlands <a href="http://www.arup.com">www.arup.com</a></p>	<p><b>ARUP</b>  <a href="mailto:brian.walker@arup.com">brian.walker@arup.com</a>  <b>Brian Walker</b> <b>44 (0) 121 213 3317</b></p>
<p><b>USA</b> Irvine, CA <a href="http://www.altair.com">www.altair.com</a></p>	<p><b>Altair Engineering Inc. Western Region</b>  <a href="mailto:info-ca@altair.com">info-ca@altair.com</a>  <b>Harold Thomas</b> <b>(949) 221-0930, ext. 206</b></p>
<p><b>USA</b> Windsor, CA</p>	<p><b>SE&amp;CS:</b>  <a href="http://www.sonic.net/lshwer/SECS/index.htm">www.sonic.net/lshwer/SECS/index.htm</a> <a href="mailto:len@schwer.net">len@schwer.net</a>  <b>Len Schwer</b> <b>(707) 837-0559</b></p>

## *Educational & Contribution Participants*

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<b>USA</b>	<b>Dr. Ted Belytschko</b>	<b>Northwestern University</b>
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<b>USA</b>	<b>Dr. David Benson</b>	<b>University of California – San Diego</b>
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<b>USA</b>	<b>Dr. Bhavin V. Mehta</b>	<b>Ohio University</b>
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<b>USA</b>	<b>Dr. Taylan Altan</b>	<b>The Ohio State U – ERC/NSM</b>
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<b>USA</b>	<b>Tony Taylor</b>	<b>Irvin Aerospace Inc.</b>
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## *Informational Websites*

<b>FEA Informational websites</b>	<b><a href="http://www.feainformation.com">www.feainformation.com</a></b>
<b>LS-DYNA Examples (more than 100 Examples)</b>	<b><a href="http://www.dynaexamples.com">www.dynaexamples.com</a></b>
<b>LS-DYNA Conference Site</b>	<b><a href="http://www.ls-dynaconferences.com">www.ls-dynaconferences.com</a></b>
<b>LS-DYNA Publications to Download On Line</b>	<b><a href="http://www.dynalook.com">www.dynalook.com</a></b>
<b>LS-DYNA Publications Index</b>	<b><a href="http://www.feapublications.com">www.feapublications.com</a></b>
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The screenshot shows the homepage of the Japan Research Institute Limited (JRI). The header features the JRI logo and name in Japanese (日本総研) and English (The Japan Research Institute Limited), along with navigation links for 'Japanese' and 'sitemap', and a search bar. Below the header is a main navigation bar with 'TOP', 'System Integration', 'Think Tank', and 'Consulting'. A large banner image shows three people in business attire, with the word 'Consulting' overlaid. To the left of the banner is a vertical menu with 'ABOUT' and 'From D Through'. Below the banner is a 'Press Release' section with a list of recent news items, including a forecast from the Bank of Japan, redevelopment plans for the Osaka Station-Umeda area, an alliance with Battelle Memorial Institute, and the release of JSTAMP-Works/NV software. The footer contains a 'Privacy Policy' and 'Contact' link, and a copyright notice for 1997-2004.

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<b>May 03</b>	<b>HP: Why HP Itanium-based workstations?</b>
	<b>Oasys and Arup Distributing &amp; Consulting</b>
<b>May 10</b>	<b>AVI: Fluid Flow Between Two Glasses</b>
	<b>DYNAFORM predicting formability problems before tooling takes place</b>
	<b>INTEL: The Intel® Itanium® 2 processor</b>
	<b>THEME: Distributor in Korea</b>
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<b>May 24</b>	<b>AVI: Contact Series</b>
	<b>NEC: NEC SX6 Series</b>
	<b>ANSYS - AI*Environment™ 2.0</b>
	<b>GissETA: Distributor in India</b>
<b>May 31</b>	<b>Website: Updated www.ls-dyna.com</b>
	<b>IBM: What is clustering</b>
	<b>Flotrend: Distributor in Taiwan</b>

<b>2004</b>	Events & Announcements
<b>Sept. 7-9</b>	<b>The Seventh International Conference on Computational Structures Technology, Lisbon, Portugal</b>
<b>Sept. 21-22</b>	<b>2004 Japanese LS-DYNA Users Conference hosted by JRI, will be held at Akasaka Prince Hotel in Tokyo.</b>
<b>Sept 21-23</b>	<b>ANSYS CHINA - Annual User Conference</b>
<b>Oct. 11-12</b>	<b>The Nordic LS-DYNA Users' Conference 2004 will be held at Quality Hotel 11, Goteborg</b>
<b>Oct.14-15</b>	<b>3rd local LS-DYNA Conference - Bamberg, Germany sponsored by DYNAmore</b>
<b>Oct. 18 - 20</b>	<b>MSC.Software's 2004 Americas Virtual Product Development Conference - October 18 - October 20 2004 Hyatt Regency Huntington Beach, CA, USA</b>
<b>Nov 10-12</b>	<b>22. CAD-FEM Users' Meeting 2004 - International Congress on FEM Technology &amp; ANSYS CFX @ ICEM CFD Conference</b>
<b>2005 &amp; 2006</b>	
<b>May 25-26, 2005</b>	<b>5th European LS-DYNA Conference - The ICC, Birmingham UK</b>
<b>June 3, 2006</b>	<b>LS-DYNA International</b>
<b>CAD-FEM GmbH announcement for two new websites:</b> <a href="http://www.lsdyna-portal.com">http://www.lsdyna-portal.com</a> <a href="http://portal.ecadfem.com/Forum.1372.0.html">http://portal.ecadfem.com/Forum.1372.0.html</a>	

## NEWSWIRE

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### **Ranking Shows More Than Half Of World's Top 500 Supercomputers Now Running On Intel® Processors**

Time, Cost Advantages of Standards-Based Building Blocks Changing the Face of Supercomputers  
SANTA CLARA, Calif., June 21, 2004 - More than half of the 500 fastest supercomputers in the world are now based on Intel® Itanium® or Xeon® processors, reflecting the trend away from deploying proprietary, one-of-a-kind supercomputers toward a building-block approach that takes advantage of standards-based, off-the-shelf components to build these powerful machines.

Intel Corporation's platforms, whose influence in supercomputing has been gaining ground rapidly in recent years, laid claim to 286 sites in the "TOP500" list of supercomputer sites released today by the University of Mannheim, the University of Tennessee and Lawrence Berkeley National Laboratory. Three years ago Intel had only three systems on the list. Intel Itanium processors are the foundation for the second-ranked new "Thunder" system at the Department of Energy's Lawrence Livermore National Laboratory (LLNL) near San Francisco and also the fastest "cluster" computer. The system took only five months to build and deploy.

Intel-based systems now hold four of the top ten spots on the list. In addition to the LLNL system, Intel processors powers systems at the National Center for Supercomputing Applications (fifth place on the TOP500), which is based on 2,500 Intel Xeon processors; Institute of Physical and Chemical Res.(seventh place), which uses 2,048 Intel Itanium 2 processors; and Pacific Northwest National Laboratory (ninth place), which employs 1,936 Intel Itanium 2 processors.

"Intel architecture's rapid rise in supercomputing reflects the acceptance of the benefits of Intel's standards-based building-block approach with its benefits of reduced design time and cost effectiveness versus the proprietary methods," said Abhi Talwalker, Intel vice president and general manager, Enterprise Platforms Group. "Using off-the-shelf components, supercomputers that used to take years to build can now be constructed in a matter of months with Intel Itanium or Xeon processors at a fraction of the cost. It's a trend that hasn't been missed by the industry as supercomputing, once the sole province of well-funded scientific pursuits, is now within the realm of a wide variety of disciplines."

Intel Itanium 2 processors have seen strong adoption in supercomputers this past year, more than tripling from 19 systems in June 2003 to 61 in the current TOP500 report. Intel Xeon processors also showed healthy growth from 100 systems a year ago to 225 in the same period.

Intel has two server architectures, which makes up approximately 85 percent of the server market segment share.\*\* The Itanium 2 processor family is targeted at business critical enterprise servers and technical computing clusters while the Intel Xeon processor family is broadly used for general purpose IT infrastructure.

### **World's Fastest Cluster Computer**

Lawrence Livermore National Laboratory's supercomputer, codenamed Thunder, took over the number two spot on the TOP500 in the new report. Configured with 4,096 Intel Itanium 2 processors, the Thunder supercomputer is world's most powerful cluster system. It is capable of 19.94 teraflops of performance. Thunder helps support LLNL's national security and science programs in fields such as inertial confinement

fusion, materials science, structural mechanics, electromagnetics, atmospheric science, biology and seismology.

"Using Intel Itanium 2 building blocks, Thunder was constructed in just five months," said Mark Seager, assistant department head at Lawrence Livermore National Laboratory, and program leader responsible for platforms. "Taking the standards-based approach with Intel components cut the time to get the system online, and satisfied a number of other factors that were critical to our needs, including price/performance, cooling, reliability and investment protection with the future processor upgrades."

The semi-annual TOP500 list of supercomputers is the work of Hans Meuer of the University of Mannheim, Erich Strohmaier and Horst Simon of the U.S. Department of Energy's National Energy Research Scientific Computing Center, and Jack Dongarra of the University of Tennessee. The complete report is available at [www.top500.org](http://www.top500.org).

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\* Other names and brands may be claimed as the property of others.

\*\* International Data Corp., (IDC), Quarterly Server Tracker, Q4, 2003.



## NORTH AMERICAN CHAPTER Your Invitation

**N**AFEMS, The International Association for the Engineering Analysis Community, is currently working on expanding its North American Chapter. With some 700 member companies worldwide, the organization is the only truly independent authority on the use of computer modelling and simulation methods, and is widely respected throughout the analysis world.




To this end, the NAFEMS North American Chapter will be holding meetings in a number of locations over the coming period. Many senior individuals from major corporations and vendors, as well as analysis specialists have already asked to be involved, and will be attending these meetings. We would like to invite you, or anyone within your company who has an interest, to attend.

The meetings are scheduled to take place at the following locations, and we invite you, or anyone within your company who has an interest, to attend, free of charge - please feel free to pass on the details of these meetings to anyone you feel may be interested. The meetings will run from 9am until 12noon on each day.






-  **Monday 19th July:** Chicago, Illinois
-  **Tuesday 20th July:** Indianapolis, Indiana
-  **Wednesday 21st July:** Detroit, Michigan
-  **Thursday 22nd July:** Boston, Massachusetts

*NB: If you are interested in participating in NAFEMS activities in North America, but cannot attend these meetings, please get in touch: if we have sufficient interest from specific areas, we will arrange further meetings in these areas at times which will suit.*

### At these meetings Tim Morris, our Chief Operating Officer, will:

-  Present a vision of the future for engineering simulation and the role of NAFEMS
-  Describe the ongoing activities of NAFEMS around the world
-  Solicit your opinions, views and suggestions for future activities.

### By attending any of these meetings, you will be able to:

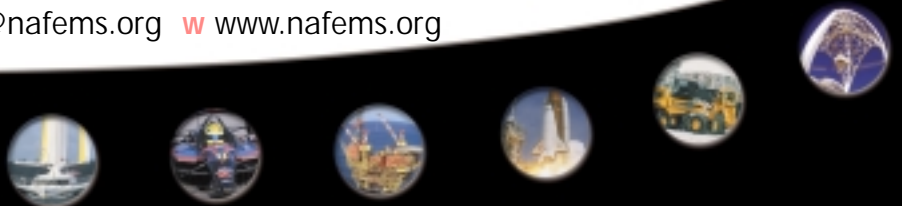
-  Gain an insight into the thoughts and strategies of some of the leading companies involved in simulation
-  Meet other like-minded individuals and discuss the future of computer modeling and simulation, putting forward your own view on the way forward
-  Improve the profile of your organization by being seen as being at the forefront of analysis technology and methods
-  Influence the activities of NAFEMS in North America and be directly involved in their delivery
-  Achieve personal and professional satisfaction from knowing that you are assisting with the development of strategies for the effective use of simulation technologies

Places at each of these meetings are still available: if you are interested in attending, please simply contact David Quinn at NAFEMS HQ, [david@nafems.org](mailto:david@nafems.org), or visit the NAFEMS website at [www.nafems.org](http://www.nafems.org) for further information.

**Remember, attendance at these meetings costs nothing**, and will provide you with the opportunity to be involved at the outset in NAFEMS activities in North America.

## Get Involved - Contact NAFEMS today

Call toll-free on 1-888-601-4207 e [info@nafems.org](mailto:info@nafems.org) w [www.nafems.org](http://www.nafems.org)





# Solving Nonlinear Equations, Part 1: Solving a Single Nonlinear Equation

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Except for the solution of dynamic problems with explicit integration in time, nonlinear finite element analyses require the solution of nonlinear algebraic equations. This chapter introduces some of the basic solution methods for solving a single nonlinear equation,

$$f(u) = 0 \tag{1}$$

where  $u$  is the unknown, and  $f(\cdot)$  is an algebraic function. All algebraic equations can be put in this form, and therefore there is no loss in generality by limiting the discussion to this form. The function is assumed to be continuous enough that its first derivative is finite. These methods are all iterative, that is, given a value of  $u$  at iteration  $i$ ,  $u_i$ , a new value is calculated for iteration  $i + 1$ ,  $u_{i+1}$ . The method is applied until the magnitude of the function is close enough to zero that the solution is acceptable. The general flow of the solution method is

```
do  $i = 1, n$ 
  if ( $|f(u)| > \epsilon$ ) then
    update  $u$ 
  else
    exit loop
  endif
enddo
```

This general algorithm requires

- The definition of  $f(\cdot)$ .
- An initial guess for the solution,  $u$ .
- A maximum number of iterations,  $n$ .
- A convergence criterion,  $\epsilon$ .

The definition of  $f(\cdot)$  for the finite element method in solid mechanics is the difference between the external forces and the internal forces. A common strategy for generating an initial guess for  $u$  is to use the solution for  $u$  from the previous time step. Notice that the iteration scheme is constructed as a do-loop which limits the number iterations to a maximum of  $n$  to prevent an infinite loop. Although some solution methods are guaranteed to converge, nothing requires them to converge in a reasonable amount of time. The convergence criterion is a simple one, but choosing the appropriate value of  $\epsilon$  is often difficult; typically it is a small fraction of the value of  $f(u)$  on the first iteration,

$$\epsilon = s \cdot f(u_1) \tag{2}$$

This, of course, begs the question of how to choose  $s$ . Like most difficult problems, the best solution is to make it someone else's problem, in other words, make it part of the required input specified by the user. Most commercial finite element codes have a heuristic strategy that picks a value somewhere between  $10^{-2}$  and  $10^{-4}$ .

## 1 Newton Iteration

Newton iteration, when it works, converges to the solution faster than any other numerical solution method in common use. It also generalizes easily to systems of algebraic equations, which isn't true for the majority of the methods. Like

The Taylor expansion of the function  $f(u)$  around the value of  $u_i$  is

$$f(u) = 0 = f(u_i) + \frac{df(u_i)}{du} \Delta u + \frac{1}{2} \frac{d^2 f(u_i)}{du^2} (\Delta u)^2 + \mathcal{O}((\Delta u)^3) \tag{3}$$

where

$$\Delta u = u - u_i. \tag{4}$$

Keeping only the linear terms gives

$$0 \approx f(u_i) + \frac{df(u_i)}{du}(u_{i+1} - u_i) \quad (5)$$

which may be solved for the updated value  $u_{i+1}$ ,

$$u_{i+1} = u_i - f(u_i) / \frac{df(u_i)}{du}. \quad (6)$$

## 1.1 Will Newton Iteration Converge?

Newton Iteration will converge provided that the initial guess,  $u_0$ , is “close enough” to the solution. Mathematics gives some insight into when the guess is close enough for convergence, but in practice, it’s simpler and more reliable to simply try it.

First, we’ll look at a generic iteration scheme, which can be written in the form

$$u_{i+1} = g(u_i). \quad (7)$$

We want to know under what circumstances  $u_i$  approaches a limiting value of  $u$ , call the *fixed point* in mathematics, as  $i$  approaches infinity. For this analysis, we’ll define converging as

$$|u_{i+1} - u_i| < |u_i - u_{i-1}|, \quad (8)$$

for all  $i > I$ , where  $I$  is some unknown, problem dependent constant. In other words, the difference between successive values of  $u$  is required to always decrease after some number of iterations. This requirement is overly strict, but it greatly simplifies the analysis. Equation 7 is substituted into Equation 8 to give

$$|g(u_i) - g(u_{i-1})| < |u_i - u_{i-1}|, \quad (9)$$

or

$$\frac{|g(u_i) - g(u_{i-1})|}{|u_i - u_{i-1}|} < 1. \quad (10)$$

As  $i \rightarrow \infty$ , the left-hand term approaches the derivative of  $g$ ,

$$\left| \frac{dg(u)}{du} \right| < 1. \quad (11)$$

For Newton Iteration,

$$g(u) = u - f(u)/\frac{df(u)}{du} \quad (12)$$

$$\frac{dg(u)}{du} = 1 - \frac{df(u)}{du} / \frac{df(u)}{du} + f(u) \frac{d^2f(u)}{du^2} / \left(\frac{df(u)}{du}\right)^2 \quad (13)$$

$$= f(u) \frac{d^2f(u)}{du^2} / \left(\frac{df(u)}{du}\right)^2. \quad (14)$$

Convergence is therefore indicated for  $u$  if

$$\left| f(u) \frac{d^2f(u)}{du^2} / \left(\frac{df(u)}{du}\right)^2 \right| < 1. \quad (15)$$

This condition says that if Equation 15 is satisfied, the next value of  $u$  calculated with Newton Iteration should be closer to the solution than the current value. There were a lot of assumptions made to arrive at Equation 15 in terms of the smoothness of the function, etc, and therefore, in practice with real-world problems, the next value of  $u$  may not be better. So while Equation 15 is interesting, it has no practical value.

On a qualitative level, Equation 15 tells us that a large gradient in the function,  $|df(u)/du|$ , helps convergence, while a large curvature,  $\sim |d^2f(u)/du^2|$ , is bad. It also reinforces the obvious, namely, being far away from the solution, indicated by a large value of  $|f(u)|$ , isn't good.

## 1.2 How Fast Will Newton Iteration Converge?

The convergence rate is how quickly the difference decreases between the current estimate of the solution and the exact solution. Assuming that the current value is sufficiently close to the solution, an estimate of the convergence rate in terms of the current error can be derived via a Taylor series.

The error at iteration  $i + 1$  is

$$e_{i+1} = u_{i+1} - u \quad (16)$$

which can also be expressed as

$$e_{i+1} = u_i - f(u_i)/\frac{df(u_i)}{du} - u \quad (17)$$

$$= u_i - u - f(u_i) / \frac{df(u_i)}{du} \quad (18)$$

$$= e_i - f(u_i) / \frac{df(u_i)}{du}. \quad (19)$$

The next two steps are unmotivated, but are necessary steps on the path to the convergence rate estimate. First, the fraction in the error expression is cleared,

$$e_{i+1} = \frac{e_i \frac{df(u_i)}{du} - f(u_i)}{\frac{df(u_i)}{du}}. \quad (20)$$

Next,  $f(u)$  is expanded in a Taylor series in terms of the error,

$$f(u) = f(u_i - (u_i - u)) \quad (21)$$

$$= f(u_i - e_i) \quad (22)$$

$$0 = f(u_i) - \frac{df(u_i)}{du} e_i + \frac{1}{2} \frac{d^2 f(u_i)}{du^2} e_i^2 + \mathcal{O}(e_i^3). \quad (23)$$

The last equation gives the approximate relation

$$\frac{df(u_i)}{du} e_i - f(u_i) \approx \frac{1}{2} \frac{d^2 f(u_i)}{du^2} e_i^2 \quad (24)$$

and, on substitution into Equation 20,

$$e_{i+1} \approx \frac{1}{2} \left( \frac{d^2 f(u_i)}{du^2} / \frac{df(u_i)}{du} \right) e_i^2 \quad (25)$$

Newton Iteration is said to converge quadratically because  $e_{i+1} \sim e_i^2$ . If this convergence rate estimate holds, close to the solution, the number of significant figures in the solution doubles with every iteration :  $e_i = 0.1$ ,  $e_{i+1} = 0.01$ ,  $e_{i+2} = 0.0001$ ,  $e_{i+3} = 0.00000001 \dots$

### 1.3 Example

Before considering problems with slow convergence, we'll look at one that behaves according to the mathematical estimates that have been developed:

$$f(u) = (u - 10)(u + 10) = u^2 - 100 \quad (26)$$

$$\frac{df(u)}{du} = 2u \quad (27)$$

$$u_{i+1} = u_i - \frac{u_i^2 - 100}{2u_i} \quad (28)$$

$$u_{\text{initial}} = 100 \quad (29)$$

$$u_{\text{exact}} = \pm 10 \quad (30)$$

The results of the Newton iteration are:

Iteration $i$	$u_i$	$e_i$	$\frac{1}{2} \left( \frac{\frac{d^2 f(u_{i-1})}{du^2}}{\frac{df(u_{i-1})}{du}} \right) e_{i-1}^2$
1	100.000000000	90.000000000	
2	50.500000000	40.500000000	40.500000000
3	26.240099010	16.240099010	16.240099010
4	15.025530120	5.025530120	5.025530120
5	10.840434673	0.840434673	0.840434673
6	10.032578511	0.032578511	0.032578511
7	10.000052896	0.000052896	0.000052896
8	10.000000000	0.000000000	0.000000000

Starting at iteration 5, the number of digits of accuracy in  $u_i$  doubles with every iteration. For all the iterations, the estimated error based on the convergence rate, Equation 25, agrees exactly with the exact error  $e_i$ . The agreement is exact because  $f(u)$  is quadratic, and all the higher order terms that were ignored in the derivation of the estimate are *exactly* zero.

## 1.4 Examples of Difficulties

Aside from a bad initial guess for  $u$ , the two primary sources of difficulties are small first derivatives and large second derivatives. Even simple functions, with derivatives of all orders, can produce convergence difficulties. Figure 1 shows Newton iteration cycling repetitively through two points forever.

The quadratic function  $f(u) = u^2 - 100$  converged very quickly in an earlier example, however, consider the simplest quadratic function,

$$f(u) = u^2 \quad (31)$$

$$\frac{df(u)}{du} = 2u \quad (32)$$

$$u_{i+1} = u_i - \frac{u_i^2}{2u_i} = \frac{1}{2}u_i \quad (33)$$

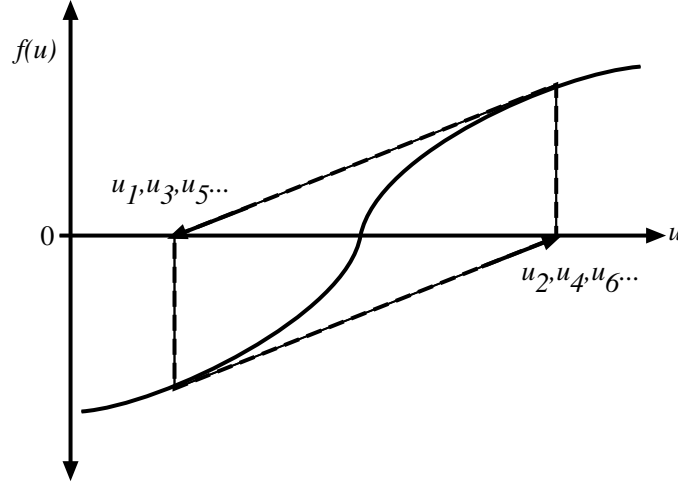


Figure 1: A smooth function that doesn't converge with Newton iteration.

$$u_{\text{initial}} = 100 \tag{34}$$

$$u_{\text{exact}} = 0 \tag{35}$$

In this case,

$$e_i = u_i - 0 = \frac{1}{2}u_{i-1} = \frac{1}{2}e_{i-1} \tag{36}$$

which shows linear, not quadratic, convergence. This occurs because the first derivative approaches zero at the same rate as the error, and is exactly zero at the solution.

Another function which exhibits poor convergence properties is the exponential,

$$f(u) = e^{\lambda u} - 1 \tag{37}$$

$$\frac{df(u)}{du} = \lambda e^{\lambda u} \tag{38}$$

$$u_{i+1} = u_i - \frac{e^{\lambda u_i} - 1}{\lambda e^{\lambda u_i}} = u_i - \frac{1 - e^{-\lambda u_i}}{\lambda} \tag{39}$$

$$u_{\text{initial}} = 10^{-2} \tag{40}$$

$$u_{\text{exact}} = 0 \tag{41}$$

For  $\lambda = 10^3$  and  $u_{\text{initial}} = 10^{-3}$ , the results are:

Iteration $i$	$u_i$	$e_i$	$\frac{1}{2} \left( \frac{\frac{d^2 f(u_{i-1})}{du^2}}{\frac{df(u_{i-1})}{du}} \right) e_{i-1}^2$
1	9.9999998E-03	9.9999998E-03	0.0000000E+00
2	9.0000452E-03	9.0000452E-03	4.9999998E-02
3	8.0001686E-03	8.0001686E-03	4.0500407E-02
4	7.0005040E-03	7.0005040E-03	3.2001349E-02
5	6.0014154E-03	6.0014154E-03	2.4503528E-02
6	5.0038907E-03	5.0038907E-03	1.8008493E-02
7	4.0106024E-03	4.0106024E-03	1.2519461E-02
8	3.0287249E-03	3.0287249E-03	8.0424660E-03
9	2.0771022E-03	2.0771022E-03	4.5865873E-03
10	1.2023950E-03	1.2023950E-03	2.1571768E-03
11	5.0286868E-04	5.0286868E-04	7.2287682E-04
12	1.0766189E-04	1.0766189E-04	1.2643846E-04
13	5.5930349E-06	5.5930349E-06	5.7955416E-06
14	1.5611900E-08	1.5611900E-08	1.5641020E-08
15	1.2186502E-13	1.2186502E-13	1.2186571E-13

Note that for the first 10 iterations,

$$u_i \approx u_{i-1} - \frac{1}{\lambda}, \quad (42)$$

and exhibits a convergence rate that is independent of the error, i.e.,  $e_i \sim (e_{i-1})^0$ . Only in the last five iterations does the asymptotic error estimate agree with the actual one, and only in the last two iterations is the quadratic convergence rate observed.

This exponential problem may seem artificial, but similar functions do occur in practice. For example, in single crystal plasticity models, a power law relates the plastic slip rate on a crystal plane to the ratio of the shear stress projected on the slip plane to a yield stress,

$$\dot{\gamma} = \dot{\gamma}_0 \left( \frac{\tau}{\tau_y} \right)^n. \quad (43)$$

The exponent  $n$  is often set to large values ( $\sim 100$ ) to create a sharply defined yield point in the stress-strain relationship. Naturally, using large values frequently creates numerical difficulties in calculations.



## 2 Secant Iteration

Secant iteration simply takes the value of  $f(u)$  at two points, connects them with a straight line, and calculates where the lines crosses zero.

$$f(u) \approx \frac{f(u_i) - f(u_{i-1})}{u_i - u_{i-1}}(u - u_{i-1}) + f(u_{i-1}) \quad (44)$$

$$u_{i+1} = u_{i-1} - \frac{u_i - u_{i-1}}{f(u_i) - f(u_{i-1})}f(u_{i-1}) \quad (45)$$

This method is cheaper per iteration than Newton iteration because it doesn't require the evaluation of the derivative of the function. With the reduction in cost comes a reduction in the theoretical convergence rate, although in practice, the observed difference is highly dependent on the problem being solved.

The secant method requires two initial values for  $u$  to start the iteration. In the absence of any insight into the problem, a typical choice for the pair of starting values is  $u_0$  and  $u_0 + \epsilon$ , where  $\epsilon$  is a small number. As the method converges, the distance between the last two successive values grows smaller, and the approximation of the derivative by the finite difference,

$$\frac{df(u)}{du} = \frac{f(u_i) - f(u_{i-1})}{u_i - u_{i-1}}(u - u_{i-1}) + f(u_{i-1}) \quad (46)$$

approaches the exact derivative, and the convergence rate approaches the rate for Newton iteration.

## 3 Methods That Are Guaranteed to Converge

The theory of continuous functions guarantees that least one solution to  $f(u) = 0$  exists between the two points  $u_L$  and  $u_R$  if

$$f(u_L) \cdot f(u_R) \leq 0. \quad (47)$$

Any algorithm that successively reduces the separation of  $u_L$  and  $u_R$  while continuing to satisfy Equation 47 will converge to a solution.

For many applications, the primary difficulty is finding an appropriate interval  $[u_L, u_R]$  for starting the iteration. However, the two common applications for this class of methods are line searches and solving for the increment in equivalent plastic strain in plasticity, and the intervals are easy to

determine. Line searches are commonly used as part of a solution strategy for systems of nonlinear equations, and the interval for the line search is usually pre-defined as  $[0, 1]$ . From continuum mechanics, the magnitude of the plastic strain increment must be somewhere between zero (an elastic solution) and the magnitude of the total strain increment.

### 3.1 The Bisection Method

The bisection method is very simple: The function is evaluated in the midpoint of the interval, and based on its sign, the midpoint becomes either the new left or right point of the interval. Written in terms of pseudo-code, the bisection method is

```

 $f_L = f(u_L)$ 
 $f_R = f(u_R)$ 
 $u = (u_L + u_R)/2$ 
do  $i = 1, n$ 
  if ( $|f(u)| > \epsilon$ ) then
     $f_{\text{avg}} = f(u)$ 
    if ( $f_{\text{avg}} \cdot f_L \geq 0$ ) then
       $u_L = u$ 
       $f_L = f_{\text{avg}}$ 
    else
       $u_R = u$ 
       $f_R = f_{\text{avg}}$ 
    endif
     $u = (u_L + u_R)/2$ 
  else
    exit loop
  endif
enddo

```

Each iteration reduces the width of the interval by a factor of two, confining the solution to an interval of length  $(u_R - u_L)/2^n$  after  $n$  iterations. While other algorithms may converge faster provided certain conditions are met regarding the derivatives, etc, the bisection method has a guaranteed convergence rate without regard to any conditions beyond the underlying continuity of the function.

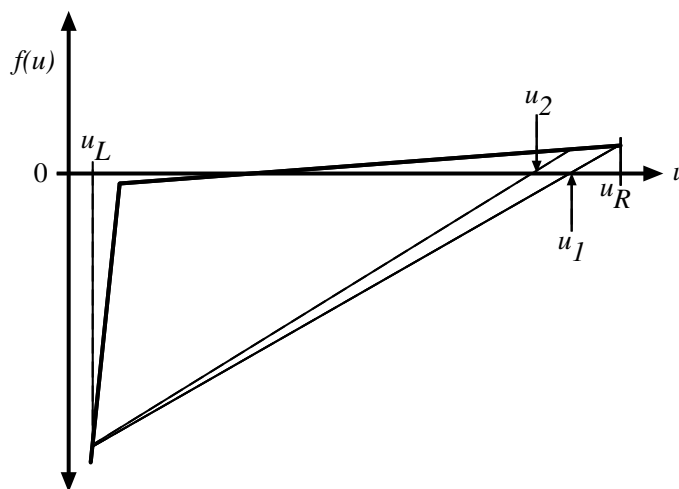


Figure 2: The first two iterations,  $u_1$  and  $u_2$  using regula falsi, demonstrating a case of slow convergence.

### 3.2 The Regula Falsi Method

Regula falsi is similar in structure to the bisection method except that  $u$  is calculated with the secant method, replacing  $u_{i-1}$  and  $u_i$  with  $u_L$  and  $u_R$  in Equation 44. If the function is well behaved, then regula falsi will converge faster than the bisection method, however it can converge very slowly for simple functions, such as the bilinear stress-strain relation for plasticity (Figure 2).

### 3.3 Ridder's Method

Ridder's method is a very robust method that works well for a wide variety of problems, including solving for the increment in equivalent plastic strain in strongly nonlinear plasticity models. The original reference is

Ridders, C. J. F., 1979, *IEEE Transaction on Circuits and Systems*, vol. CAS-26, pp. 979-980.

however it was popularized in the very popular book *Numerical Recipes: The Art of Scientific Computing*. This book is available with its programs written in Fortran, C, and C++, and the original editions in Fortran and C are available on-line as PDF files at <http://www.library.cornell.edu/nr/>.

The function is evaluated two times for every iteration. The first evaluation is at the midpoint of the interval,

$$u_M = \frac{1}{2}(u_L + u_R) \quad (48)$$

$$f_M = f(u_M) \quad (49)$$

The location of the solution is a function of the two endpoints and the midpoint,

$$u = u_M + (u_M - u_L) \frac{\text{sign}|f_L - f_R|f_M}{\sqrt{f_M^2 - f_L f_R}}. \quad (50)$$

The standard method updates the interval using  $u$  in the same manner as the bisection and regula falsi methods. A simple modification used by the author which seems to significantly increase the robustness of the method and slightly improve the speed of convergence is to perform a second update on the interval using  $u_M$  if  $f(u_M) \cdot f(u) < 0$ .