# Increasing reliability of metal forming processes in early design stages

Stephan Pannier[1], Kathrin Grossenbacher[2], Martin Liebscher[3], Markus Ganser[2],
Wolfgang Graf[1], Heiner Müllerschön[3], Arnulff Lipp[2], Michael Kaliske[1]
[1]Institute for Structural Analysis, Technische Universität Dresden, 01062 Dresden, Germany
[2]BMW AG, Knorrstr. 147, 80788 Munich, Germany
[3]DYNAmore GmbH, Industriestr. 2, 70565 Stuttgart, Germany

**Summary:**

The identification of an appropriate design for engineering structures and processes is a development task over a long period of time. Finally, the developed design should be primarily reliable. Focusing on reliability purposes just in an advanced design stage may necessitates modifications which are cumbersome to realize and expensive additionally. Hence, the assessment of reliability demands has to be set in at the very beginning of the design process. Thereby, the needed information to assess the reliability in a customary way is missing in general. Thus, a first measure requires to model uncertainty on the basis of available information, which originates in early design stages from experience and expert knowledge, to determine reliability statements. This is done reasonably with the aid of the uncertainty characteristic fuzziness. Thus, a reliability analysis is performed by means of a fuzzy structural analysis. In order to keep those procedures applicable for computational expensive applications, which is always a demanding task in reliability analysis, the fuzzy results are determined approximatively.

The results of the reliability analysis provide not only the basis for comparative evaluations, but rather should be utilized to infer design specifications for the further design process. On account of intended reliability requirements the resulting point sets can be subdivided into permissible and non-permissible points. The aim is to abandon non-permissible regions from the design space and determine alternative design spaces appropriately. On account of the fact, that interaction between input parameters should be avoided, those alternative design spaces are described by means of hypercuboids. The functional dependencies between input and result parameters can not be assumed to be one-to-one in advance. Thus, the alternative input spaces can be non-connected. This is taken into account by solving an inverse problem. In this paper, a cluster analysis approach is applied. On the basis of the cluster results, the announced alternative design spaces are specified by means of hypercuboids. Those alternative design spaces provide the basis to update the design in view of reliability requirements in an early design stage. In general, the hypercuboids should be as large as possible to provide a latitude for the further design process, but should be also well-founded on available permissible points.

The presented approach, which combines multiple innovative solution statements, provides a tool to affect the design process in early stages in view of reliability requirements while leaving scope for development, which is of prime importance for multidisciplinary design tasks. Thereby, the applicability of the presented approach is demonstrated be means of an industry-relevant example.

**Keywords:**

inverse solution, fuzzy analysis, cluster analysis, alternative design spaces

## 1 Introduction

The design of engineering structures and processes presupposes most often a cooperation of many engineering disciplines over a long-lasting period of time. Thus, the identification of the final design is a highly complex task, which cannot be mapped in a closed simulation run. Nevertheless, the design has to fulfill multiple reliability requirements in the very end. Please note, that constraints in a deterministic design task become reliability requirements when uncertain input parameters are incorporated. On account of the numerical expense, the consideration of uncertainty within the input parameters and, in consequence, the assessment of reliability is usually done in a final design stage. This is additionally advanced by the unavailability of information to specify those input parameters appropriately. Contrarily, the modification of a design in an advanced stage will become cumbersome and expensive because of the ambitiousness to satisfy all disciplines with the modified design. Hence, the incorporation of reliability considerations in an early design stage is a consistent measure to further improve the design process of engineering structures and processes.

In the early design stage the conditions differ from those in a final stage. The fundamental and, thus, most important specifications to characterize the design are made. This means, detecting problems with the intended design in the whole can be remedied with low efforts. However, due to the complexity of the design task it has to be avoided to limit the scope for decisions within the design process rigorously.

The aim of the targeted approach is to determine design ranges and uncertainty ranges of parameter values, which fulfill reliability requirements in a best possible manner. On account of the information deficit in those early design stages the reliability cannot be guaranteed but should be rather improved. Generally, the modification on the predefined ranges should be as few as possible. In conclusion, the aim is not to determine an optimal design in his strict sense, but to sort out unwanted possible designs.

A delicate subject in this approach is an appropriate modeling of the uncertain information available. In a general definition, it is distinguished between aleatoric and epistemic uncertainty [2]. While aleatoric uncertainty may be interpreted as randomness and modeled with stochastic variables, epistemic uncertainty arises from subjective assessments of objective data. To model epistemic uncertainty, a variety of uncertainty models are on hand [7]. In an early design stage of the design process, the available information and data are limited, rare, imprecise and vague. Furthermore, they base on expert specifications and experience. Therefore, the introduction of epistemic uncertainty models is most reasonable, i.e., the uncertainty model fuzzy, based on the fuzzy set theory [11, 6, 7], is introduced. Thereby, it is dispensable to generate unwarranted information by compelling available data in a stochastic model.

An application of a plain optimization algorithm just focuses on a single optimal design. Beside the fact, that the design is fixed too rigorous to some parameter values, it is impossible to consider the present uncertainty and thus to consider reliability aspects. Contrarily, the plain application of a reliability assessment enables to evaluate the quality of the actual design appropriately, but gives no hints about the required design update. The combination of both approaches within a reliability based optimization may be a versatile tool at all, but also misplaced for the design in an early design stage. Reasons therefore are the high numerical expense, problems to specify stochastic input quantities appropriately and the rigidity of the fixed design.

On the basis of a reliability assessment, measures are required to sort out design ranges and uncertainty ranges of parameters, which does not fulfill the intended reliability requirements. Thereby, the aim is to reduce the amount of modifications to an essential extend due to the direct proportion to costs. This is done by detecting alternative, reliable design spaces on the basis of an inverse solution approach.

In this paper, a novel approach for a reliability based design update is presented, which just utilize available information and returns a feasible design update with a minimal request on modification. The approach is tailored to the industrial application of deep drawing processes. The applicability is demonstrated by means of an industry-relevant problem.

## 2 General approach

The general approach is illustrated for the design of a sheet metal forming process. The aim is to determine a manufacturing process, which meets the specified criteria on the geometry in a best possible manner and, additionally, produces as few rejects as possible. Thereby, the avoidance of rejects, which is linked with a high reliability level, will reduce the costs of manufacturing noticeable.

In order to model design variables and a priori parameters simultaneously [5], it is assumed, that all varying parameters can be interpreted as uncertain quantities. It is indisputable, that the modeling of this kind of uncertainty with stochastic quantities is unwise. In this approach, the application of fuzzy sets is preferred. The presence of fuzzy quantities in input parameters enforces to apply fuzzy structural analysis. If the reliability fulfills user-specified requirements, the design can remain unaffected. Otherwise the design should be modified utilizing the results of the reliability calculation.

A decision about the modification of the design should be drawn on the basis of the reliability results. Hence, the determined points in the fuzzy structural analysis provide the design of experiments for the design update. To sort out non-permissible design regions, the task is to detect permissible, alternative design regions and specify them appropriately. For the purpose of setting the lowest demands onto the updated design ranges, interaction free hypercuboids are applied. The hypercuboids should represent the respective point set in a best possible manner. Thus, detecting alternative design ranges is an optimization procedure itself.

In a design update procedure the alternative design ranges are determined on the basis of initial point sets. This is important, if the functional relationship between input and result parameters is just unique. Then, a distinct assignment of alternative design ranges is not possible and disconnected permissible ranges in the input space have to be detected. This may be done by means of classification procedures, i.e., cluster analysis.

## 3 Uncertainty quantification and processing with imprecise data

The quantification of uncertainty in early design stages is cumbersome due to the absence of information. Since a description with random quantities is misplaced, epistemic uncertainty models are more appropriate, because they enable to introduce subjective assessments like experience and expert knowledge. A simple epistemic uncertainty model for continuous input parameters utilizes intervals [8]

$$u = [l, r] = \{a \in \mathbb{R};\, l \leq a \leq r\}\ . \tag{1}$$

The aim of uncertainty quantification is to model all available information as realistic as possible. The demand of a realistic quantification prohibits the application of stochastic quantities. But to just introduce interval quantities will skip a huge amount of available information. Therefore, fuzzy quantities are applied. Fuzzy quantities enable to assess an uncertain range additionally with a membership grade. Hence, a subjective weighting of available information is enabled. A (normalized) fuzzy set $\tilde{u}$ is defined by a membership function $\mu_{\tilde{u}}$, which assesses each $a \in \mathbb{R}$ with a membership grade $\mu_{\tilde{u}}(a)$. Hence, $\mu_{\tilde{u}} = \tilde{u}$ is defined as the mapping of the real line to the interval $[0, 1]$

$$\tilde{u} : \mathbb{R} \rightarrow [0, 1]\ . \tag{2}$$

A n-dimensional fuzzy quantity $\tilde{u}_n$ is defined as

$$\tilde{u}_n : \mathbb{R}^n \rightarrow [0, 1] : (x_1, \ldots, x_n) \mapsto \min\left(\mu_{\tilde{u}}(x_1), \ldots, \mu_{\tilde{u}}(x_n)\right)\ . \tag{3}$$

In view of numerical realization, a fuzzy number can be represented as a family of $\alpha$-level-sets

$$\tilde{u} = \left(u_\alpha; \alpha \in (0, 1]\right)\ . \tag{4}$$

This representation is denoted as $\alpha$-level-discretization. Thereby, each $\alpha$-level-set $u_\alpha$ is an interval as defined in Eq. 1

$$u_\alpha = \{x \in \mathbb{R}; \mu_{\tilde{u}}(x) \geq \alpha\}\ . \tag{5}$$

 

In general convex fuzzy numbers are applied, even though non-convex fuzzy numbers are still under development [10]. A fuzzy number is convex, if each $\alpha$-level-set $u_\alpha$ is a convex set and for $\lambda \in [0,1]$ and any $x_1, x_2 \in \mathbb{R}$

$$\mu_{\tilde{u}}\left(\lambda x_2 + (1-\lambda)x_1\right) \geq \min\left(\mu_{\tilde{u}}\left(x_1\right) \mu_{\tilde{u}}\left(x_2\right)\right) \tag{6}$$

holds. Introducing fuzzy quantities within the structural analysis expands the task to be solved to a fuzzy structural analysis. For a numerical simulation of a fuzzy structural analysis, the problem is discretized to a point-to-point evaluation of the structural analysis $f$. A structural analysis

$$f: \quad \mathbb{R}^n \to \mathbb{R}: \quad x \mapsto z = f(x), \tag{7}$$

bases for example on the solution of a system of partial differential equations. Thus, a fuzzy structural analysis $f^F$ may be formulated with the aid of the extension principle

$$\begin{aligned} f^F: \quad \mathcal{F}\left(\mathbb{R}^n\right) \to & \quad \mathcal{F}\left(\mathbb{R}\right) \\ \tilde{x} \mapsto & \quad \tilde{z} := \left( z \mapsto \sup_{a \in f^{-1}(\{z\})} \min\left(\mu(a_1), \ldots, \mu(a_n)\right) \right), \end{aligned} \tag{8}$$

with $\mathcal{F}(.)$ indicating a set of fuzzy numbers. A numerical realization of Eq. 8 is cumbersome in application. Alternatively, the fuzzy structural analysis can be solved by means of an $\alpha$-level-optimization approach. The basis of this approach is the $\alpha$-level-discretization of fuzzy quantities. Thereby, the input quantities are mapped to result quantities $\alpha$-level wise. The aim is to determine for each input interval $x_\alpha$ the respective result interval $z_\alpha$, i.e. the minimal and maximal possible result values $z_{\alpha,l}$ and $z_{\alpha,r}$.

$$f_\alpha^F: \quad \{[a,b]; a \leq b; a, b \in \mathbb{R}\}^n \to \{[c,d]; c \leq d; c, d \in \mathbb{R}\}: x_\alpha \mapsto z_\alpha := [z_{\alpha,l}, z_{\alpha,r}] \tag{9}$$

$$\text{with} \qquad z_{\alpha,l} = \min_{x \in x_\alpha} f(x) \quad z_{\alpha,r} = \max_{x \in x_\alpha} f(x) \tag{10}$$

To evaluate $z_{\alpha,l}$ and $z_{\alpha,r}$ numerically, optimization procedures are applied.

On the basis of the determined fuzzy result parameters $\tilde{z}$, the reliability of the structure can be assessed. Therefore, various approaches are available [6]. A crude way of assessing the reliability provides a worst case consideration. Thereby, either all possible result values fulfill the reliability requirements or even not. For the latter, measures to increase the reliability of the design have to be formulated. Therefore, anti-optimization approaches [9] can be applied.

For the application of fuzzy structural analysis some recommendations can be made in view of the deep drawing process under consideration. From a theoretical point of view, for an infinite number of $\alpha \in [0,1]$ Eq. 10 has to be solved; the inapplicability is obvious. To characterize the result quantity appropriately, five $\alpha$-levels seem to be enough for a variety of applications. Nevertheless, ten optimization tasks have to be processed, which is far beyond the available computational capabilities. A further simplification utilizes the fact, that we primarily are not interested in the fuzzy result $\tilde{z}$ itself, but rather on a reliability assessment. In example, this fact is utilized in stochastic structural analysis, when variance reduction methods are applied. However, it is dispensable to determine all $z_{\alpha,l}$ and $z_{\alpha,r}$, generally either $z_{\alpha,l}$ or $z_{\alpha,r}$ have to be evaluated. Furthermore, concentrating on a lower number of $\alpha$-level-cuts will speed up the efficiency additionally. We recommend to evaluate $z_{\alpha=0,l}$ or $z_{\alpha=0,r}$ alternatively, while an arbitrary point $p \in x_{\alpha=1}$ should be used as start point of the optimization algorithm. Thus, an approximated fuzzy result $\tilde{z}^*$ is obtained.

## 4 Detection of alternative design spaces

Most of research in engineering design focuses on the detection of an optimal design. Therefore, sophisticated methods like reliability based and robustness based design procedures are on hand. The approach of detecting alternative design spaces is not common but essential in early design stages. As mentioned above, the aim is to eliminate non-permissible regions in the design space and determine proper alternative design spaces.

The introduced approach can be subdivided into four main parts (see also Fig. 1).

- Generate an appropriate design of experiments (DoE). In the presented approach, the results of the fuzzy structural analysis $f^F$ provide a point set $\mathcal{H} = \left((x,z)_1, \ldots, (x,z)_{n_{sim}}\right), x \in \mathbb{R}^n, z \in \mathbb{R}^m$, which can be interpreted as a DoE. The number $n_{sim}$ of points is ruled by the required simulations runs for the evaluation of $f^F$. The support of the fuzzy input quantity $spt\,(\tilde{x})$ defines the initial input space $S$.
- Formulate limits and constraints for the design problem. Assigning the limits and constraints to $(x_1, \ldots, x_{n_{sim}})$ or $(z_1, \ldots, z_{n_{sim}})$, the points are grouped into permissible $\mathcal{H}_{perm}$ and non-permissible $\mathcal{H}_{nop}$ ones. This process can be characterized as a part of a reliability assessment.
- Partition the points $\left(x_1, \ldots, x_{n_{perm}}\right) =: \mathcal{H}^x_{perm}$, extracted from $\mathcal{H}_{perm}$, appropriately. On account of the inverse assignment $\left(f^{-1}(z_1), \ldots, f^{-1}(z_{n_{perm}})\right)$ of unique functions, the points $\left(x_1, \ldots, x_{n_{perm}}\right)$ have to be classified in order to detect non-connected input spaces. In this approach, cluster analyses are applied.
- Specify alternative design spaces. The alternative design spaces are constituted on the basis of $\mathcal{H}^x_{perm}$ and the detected cluster configuration. Generally, those alternative design spaces have an arbitrary shape. Reasonably, the design spaces are modeled with hypercuboids. Thus, the interaction between input parameters can be neglected in the further design process.

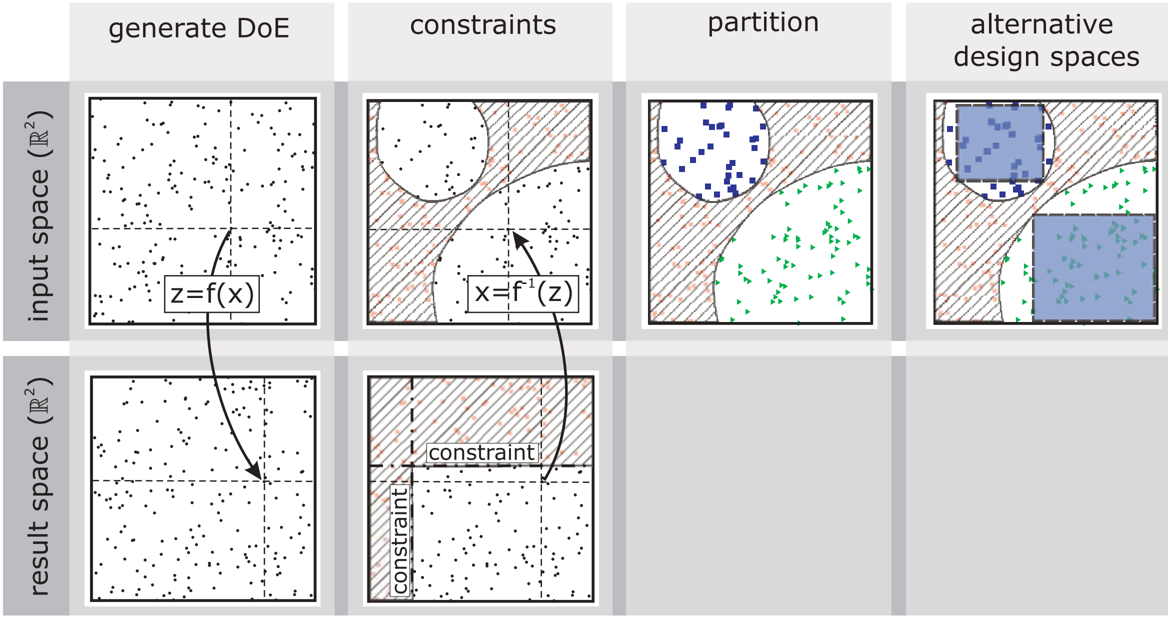


*Figure 1:* Scheme for detection of alternative design spaces

### 4.1 Cluster analysis

The cluster analysis approach is a data mining method. The aim is to determine data structures within a predefined point set. Such a structure may originate from the generation of a DoE or from an inverse assignment of preselected points, like in this approach. This problem can be solved for low dimensions (up to three) by using 3D-scatterplots. But for higher dimensions, the characterization of clustered points by means of graphical tools is cumbersome. Therefore, numerical approaches are required.

The idea in this approach is to apply cluster analysis in order to partition $\mathcal{H}^x_{perm}$ into $n_c$ subsets (clusters) $\mathcal{C}$. Thereby, the set of all $\mathcal{C}_i$, $i = 1, \ldots, n_c$ is denoted as cluster configuration $K$, while generally an arbitrary set of cluster configuration $\mathcal{O}$ is present

$$\mathcal{O} := \left\{ K \subseteq P^{\left(\mathcal{H}^x_{perm}\right)} \right\} . \tag{11}$$

For each cluster configuration $K$, the respective subsets $\mathcal{C}_i, \mathcal{C}_j \in K$ are pairwise disjoint, nonempty and reproduce $\mathcal{H}_{perm}$ [1]. Those three conditions are expressed by

$$\mathcal{C}_i \cap \mathcal{C}_j = \varnothing, \quad \mathcal{C}_i \neq \mathcal{C}_j \tag{12}$$

$$\mathcal{C}_i \neq \varnothing \tag{13}$$

$$\bigcup_i^{n_c} \mathcal{C}_i = \mathcal{H}^x_{perm} \tag{14}$$

and hold for all $\mathcal{C}_i, \mathcal{C}_j \in K$. The points within a cluster $\mathcal{C}_i$ should be as homogeneous as possible and the points between different clusters $\mathcal{C}_i, \mathcal{C}_j, i \neq j$ should be as heterogeneous as possible. Thereby, homogeneity characterizes the resemblance of arbitrary points $p, q \in \mathcal{C}_i$ within in the same cluster. One way to assess the resemblance of points provides the analysis of the distance $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$. Thereby, the resemblance increases, when the sum of the distances between all points becomes minimal

$$\sum_{p,q \in \mathcal{C}_i} d(p,q) \to \min \ . \tag{15}$$

Heterogeneity is the counterpart to homogeneity and, hence, assessed with a distance measure. Thereby, a high heterogeneity between points is present, when the sum of the distances between all points is maximal

$$\sum_{p \in \mathcal{C}_i, q \in \mathcal{C}_j, i \neq j} d(p,q) \to \max \ . \tag{16}$$

The objective of the cluster analysis is to determine a cluster configuration $K$ in such a manner, that for all clusters $\mathcal{C}_i \in K$ the sum of the respective squared distance between each point of the cluster $p \in \mathcal{C}_i$ and the empirical mean $\nu_i$ of cluster points is minimal

$$D(K) \to \min \tag{17}$$

$$\text{with} \quad D: \ \mathcal{O} \to \mathbb{R}: \ K \mapsto \sum_{\mathcal{C}_i \in K} \sum_{p \in \mathcal{C}_i} d(p, \nu_i)^2 \ . \tag{18}$$

In extension of this deterministic classification procedure, a fuzzy classification procedure can be applied. Thereby, the membership $\mu_{\mathcal{C}_i}(p)$ of points $p \in \mathcal{C}_i$ is not expressed in a binary manner $\{0,1\}$, but rather in a continuous range $[0,1]$. Thus, the task to be solved reads

$$D^F(K) \to \min \tag{19}$$

$$\text{with} \quad D^F: \ \mathcal{O} \to \mathbb{R}: \ K \mapsto \sum_{\mathcal{C}_i \in K} \sum_{p \in \mathcal{C}_i} \mu_{\mathcal{C}_i}(p)^w \cdot d(p, \mu_i)^2 \ , \tag{20}$$

where $w$ is a weighting exponent.

For the solution of the classification problem, three user specifications are required: the number of clusters $n_c$, the cluster initialization and a distance metric [3]. Determining the accurate number of cluster is an optimization task itself. It presents the greatest challenge for the application of cluster analysis so far. A measure varies $n_c$ in user-specified ranges and assesses the respective appropriateness by means of quality measures. However, in [1] three distance measures $d$ are introduced, though many more can be formulated.

### 4.2 Alternative design spaces

The results of a cluster analysis are point sets $\mathcal{C}_1, \ldots, \mathcal{C}_{n_c}$ with an arbitrary spreading. Opposite to other classification methods, e.g., supporting vector regression, the results of a cluster analysis give no hints about the boundary between neighbored clusters. Here, this is not worthwhile anyway, because the boundaries between the clusters itself are of no importance. Rather the focus is set on detecting the boundaries between $\mathcal{C}_i \in \mathcal{H}^x_{perm}$ and $\mathcal{H}^x_{nop}$ $\left(\mathcal{H}^x_{perm} \cup \mathcal{H}^x_{nop} = (x_1, \ldots, x_{n_{sim}})\right)$. Obviously, on the basis of a restricted initial point set the boundary can be determined in an approximate manner. Nevertheless, assigning a convex hull to a point set $\mathcal{C}_i \in K$ provides possibly the best specification of the alternative design space. But the applicability of such a convex hull is extremely limited, because interactions have to be considered between the input parameters. Thus, hypercuboids, which enable to describe input parameters under neglect of interactions, are applied. An assigned hypercuboid $H_i^n \subset S$ is a subset of the initially defined input space $S$ and should incorporate all points $x \in \mathcal{C}_i$ of a specific cluster

$$\forall x \in \mathcal{C}_i : \quad x \in H_i^n \; . \tag{21}$$

This condition is preferable but not a prerequisite. To define a hypercuboid in the first attempt two opposite vertices $v^{min}, v^{max}$ may be determined on the basis of available points $x \in \mathcal{C}_i$ by

$$v^{min} = (\min(x_{1,1}, \ldots, x_{1,ppc_i}), \ldots, \min(x_{n,1}, \ldots, x_{n,ppc_i})) \tag{22}$$

$$v^{max} = (\max(x_{1,1}, \ldots, x_{1,ppc_i}), \ldots, \max(x_{n,1}, \ldots, x_{n,ppc_i})) \quad . \tag{23}$$

The quantity $ppc_i$ indicates the points per cluster $\mathcal{C}_i$. A prerequisite on the alternative design space, represented with the hypercuboid, is the validity. Hence, it has to be guaranteed, that no non-permissible point out of $x \in \mathcal{H}^x_{nop}$ is located in the hypercuboid

$$H_i^n \cap \mathcal{H}^x_{nop} = \varnothing \; . \tag{24}$$

Generally, two different approaches are available to assign hypercuboids to the clusters. First, the hypercuboid determined in Eq. 21 describe the maximal possible hypercuboid and may shrink under application of Eq. 24. This is appropriate to assign proper hypercuboids to individual clusters $\mathcal{C}_i \in K$. Thereby, the volume of the hypercuboid is ruled by the cluster design. Second, the hypercuboid in Eq. 21 provides just the basis and expands steadily under regard of Eq. 24. In result, the hypercuboid span eventually over multiple clusters simultaneously. Ideally, the assigned hypercuboids are identical for all cluster configurations $K \in \mathcal{O}$. The objective is to constitute a hypercuboid with a maximal possible volume

$$\sup \left\{ V_{H_*^n}; H_*^n \cap \mathcal{H}^x_{nop} = \varnothing, H_*^n \subseteq S \right\} \tag{25}$$

$$\text{with} \quad V_{H_*^n} = \left( \left( v_1^{max} - v_1^{min} \right) \cdot \ldots \cdot \left( v_n^{max} - v_n^{min} \right) \right) \; . \tag{26}$$

On account of the supremum formulation, the hypercuboid of interest $H_i^n$ is determined with condition $|V_{H_*^n} - V_{H^n}| < \epsilon$ for $\epsilon > 0$. Supplementary, additional preferences should be introduced to account for the fact, that it is worthwhile to modify a few parameter values extensive instead of modifying all parameter values moderately. Generally, as far as Eq. (22) is fulfilled, the hypercuboid is classified as valid. In result, the determined hypercuboid does not guarantee an intended reliability level, it just represents an improved alternative design space in view of reliability requirements.

## 5 Examples

### 5.1 Rosenbrock function

In this example the introduced approach should be visualized with the aid of the Rosenbrock function (see Fig. 2)

$$f(x, y) = (1 - x)^2 + 100 \cdot \left( y - x^2 \right)^2 \quad x \in [-2, 2], \; y \in [-1, 3] \quad . \tag{27}$$

In general, this problem can be solved manually. Here, it should just show the principal idea of the presented approach. The aim is to detect alternative design regions for result values $z > 500$. First of all, the input parameters are modeled as fuzzy quantities, in this example the special case of intervals is applied (see Eq. (27)). The fuzzy analysis is performed introducing a multi-chain philosophy in the $\alpha$-level optimization procedure. This enables to reveal different regions of extremal points. In total 120 points are evaluated. Obviously, reliability problems occur, which requires the determination of alternative design spaces.

The 120 points are subdivided into permissible $z_{perm} = \{z; z > 500\}$ and non-permissible $z_{nop} = \{z; z < 500\}$ points. For the permissible points $z_{perm}$ a cluster analysis is applied. Thereby, $n_c = 1, \ldots, 5$ clusters are evaluated and assessed. As expected, a cluster configuration with $n_c = 3$ represents the

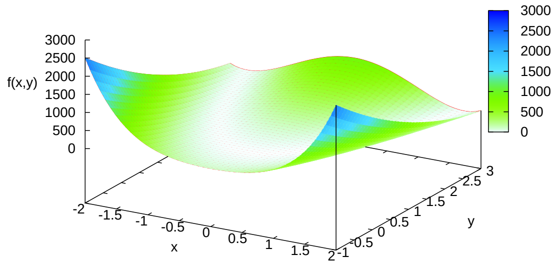


*Figure 2:* Rosenbrock function

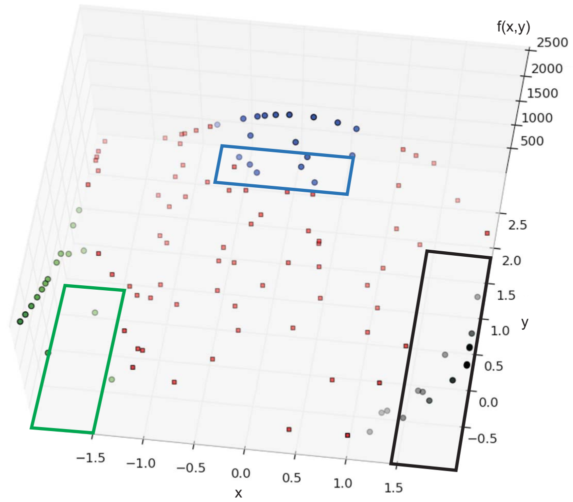


*Figure 3:* Three non-connected alternative design spaces for $z > 500$

structure within the input quantities in the best possible manner. In Fig. 3 the red dots show the non-permissible results, the black, green and blue dots represent the points of different clusters respectively. In order to provide alternative design spaces, hypercuboids are assigned to the clusters (see also Fig. 3). Thereby, maximal hypercuboids are determined in accordance to Eq. (22) and (23). In this example the hypercuboids just shrink under consideration of Eq. (24). In result, on the basis of a given point set, three alternative design spaces are provided which just contain permissible points. Note that, even though the rosenbrock function is symmetric, the respective alternative design spaces are not (green and black rectangles in Fig. 3), due to an asymmetric DoE.

### 5.2 Design of a deep drawing process

To demonstrate the capabilities of the introduced approach, an industry-relevant application should be examined. Here, a surrogate problem is solved, which features all occurring characteristics of an industrial application. The main advantage is to consider shape modifications be means of parameterized models. Within the example the whole process is simulated; starting point is the deep drawing, followed by the trimming and finally analyzing the springback.

The model, see Fig. 4, consist of 4 tubes, which represent the punch and the die, the blank and some line forces to represent the draw beads. Thereby, some draw bead forces are arranged outside of the blank to take into account even insensitive parameters. In total 28 input parameters are present. These are the radius of the two dies, 22 draw bead forces, shell thickness, binder force and the initial positioning of the blank in both directions. Details about those parameters are shown in Fig. 6.

The objective of this investigation is to minimize the deviation from the intended geometry. This is numerically evaluated by means of the sum of absolute differences $z$ between the geometry after springback and the intended geometry, evaluated in nodes of the finite element mesh. Such an evaluation is depicted in Fig. 5. In order to comply with requirements on the final geometry, the objective is additionally constraint. Furthermore, constraints are introduced to ensure a reliable manufacturing process. First of all, to be consistent in the forming limited diagram, the cracking is evaluated in all discretized points of the blank. The cracking value is defined as the considered strain state, normalized with the forming limit curve (FLC). By definition, if these parameters exceed a value of $1.0$, the deep drawing device should be rejected. Second, it has to be ensured, in view of a robust manufacturing process, that the edge of the blank does not pass the draw beads.

On the basis of available information the input parameters are modeled as fuzzy triangular quantities, see Fig. 7, which can be noted with $< x_{\alpha=0,l}, x_{\alpha=1}, x_{\alpha=0,r} >$. Taking into account, that one solver run of this model needs 7-9 hours on a single CPU, the fuzzy quantities are just discretized on two

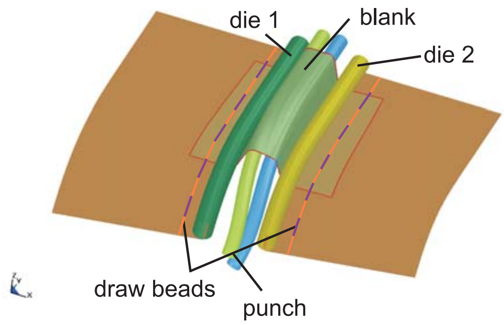


*Figure 4:* Model of deep drawing process

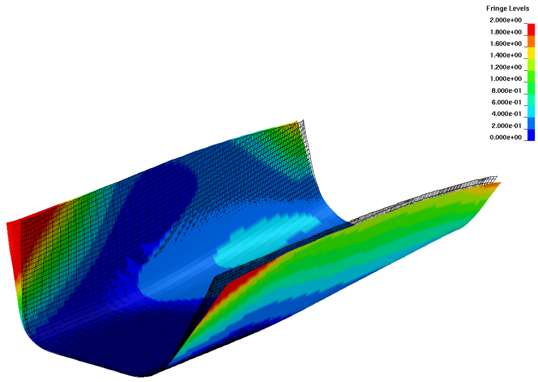

*Figure 5:* Result of springback simulation (displacement from intended geometry evaluated)

$\alpha$-levels. Thereby, especially the results in non-permissible regions are of prime importance, thus, the determination of the respective minimum is neglected. In result, just 232 points were simulated to approximate the fuzzy result quantity $\tilde{z}$ (see Fig. 6).

| input parameter | ranges | fuzzy quantities |
|---|---|---|
| radius die 1 | $8 \dots 12$ | $< 8, 10, 12 >$ |
| radius die 2 | $8 \dots 12$ | $< 8, 10, 12 >$ |
| draw bead force 1 | $0 \dots 300$ | $< 0, 200, 300 >$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| draw bead force 22 | $0 \dots 300$ | $< 0, 200, 300 >$ |
| shell thickness | $0.45 \dots 0.5$ | $< 0.45, 0.475, 0.5 >$ |
| binder force | $100 \dots 300$ | $< 100, 200, 300 >$ |
| positioning blank x-direction | $-2 \dots 2$ | $< -2, 0, 2 >$ |
| positioning blank y-direction | $-2 \dots 2$ | $< -2, 0, 2 >$ |

*Figure 6:* Input parameters

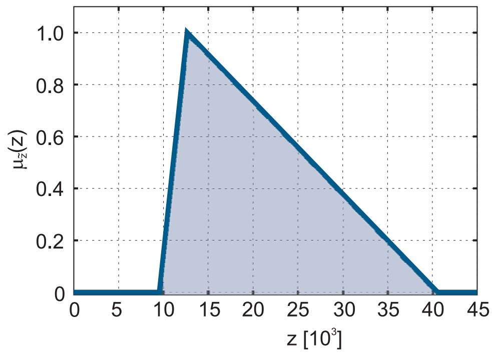


*Figure 7:* Fuzzy result $\tilde{z}$

The 232 points of the fuzzy analysis provide the initial point set $\mathcal{H}$ (DoE) to determine alternative design spaces. The evaluation of the constraints enable the detection of $\mathcal{H}^z_{perm}$ and via an inverse assignment $\mathcal{H}^x_{perm}$. For the point set $\mathcal{H}^x_{perm}$, a cluster analysis is applied. A classification up to four clusters ($n_c = 4$) is investigated to verify, if non-connected input regions exist. Because a visualization of the results is impossible, here, one cross-plot for three clusters is shown in Fig. 8. Thereby, the red dots indicate non-permissible points and the blue, green and black dots indicate points of the respective clusters. Obviously, to determine a structure with the help of multiple of such cross-plots is impossible. Furthermore, a decision about the best cluster configuration should be drawn, which can be realized just by evaluating quality measures (see [4]). Thereby, low values to assess the homogeneity (Eq. (15)) and high values to assess the heterogeneity (Eq. (16)) are in favor. In result, a suggestion for a appropriate cluster configuration is obtained. Nevertheless, an indication, that the whole input space is connected eventually, is not provided by those measures.

Hypercuboids are assigned to the respective clusters to enable a proper description of the alternative design spaces. The result is visualized in Fig. 9 for the cluster configuration depicted in Fig. 8. Checking all other input dimension it can be observed, that the boxes overlap in all possible cross-plots. Thus, the permissible input region seems to be connected. Note, that this statement is founded on a sparsely set of 232 points. The selected alternative design space coincides almost with the green box in Fig. 9 and the coordinates are given in detail in Fig. 10.

 

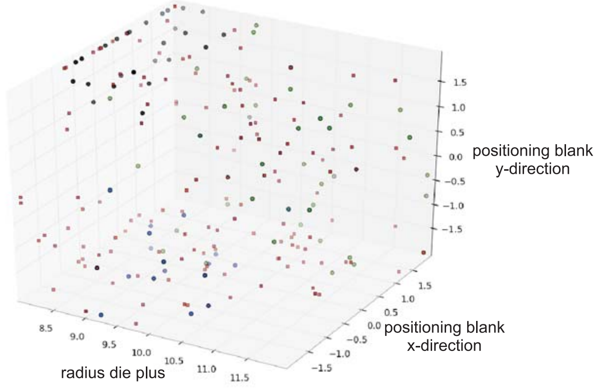


*Figure 8:* Result of cluster analyis

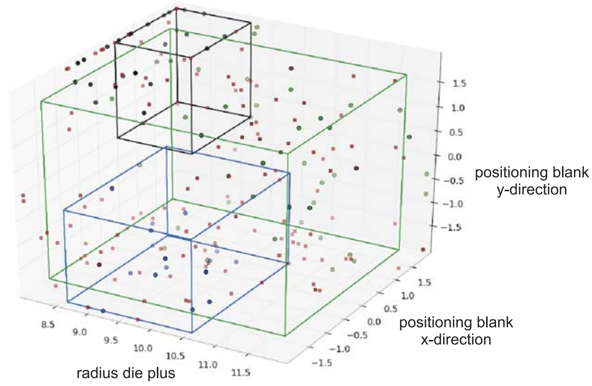


*Figure 9:* Alternative design spaces

| input parameter | coordinates hypercuboid |
|---|---|
| radius die 1 | $[9.3, 9.9]$ |
| radius die 2 | $[9.05, 12.0]$ |
| draw bead force 1 | $[0.0, 300.0]$ |
| ⋮ | ⋮ |
| draw bead force 15 | $[0.0, 300.0]$ |
| draw bead force 16 | $[20.2, 300.0]$ |
| draw bead force 17 | $[0.0, 300.0]$ |
| draw bead force 18 | $[12.65, 300.0]$ |

| input parameter | coordinates hypercuboid |
|---|---|
| draw bead force 19 | $[0.0, 300.0]$ |
| ⋮ | ⋮ |
| draw bead force 22 | $[0.0, 300.0]$ |
| shell thickness | $[0.469, 0.476]$ |
| binder force | $[100.0, 292.0]$ |
| positioning blank x-direction | $[-2.0, 1.24]$ |
| positioning blank y-direction | $[-1.6, 0.8]$ |

*Figure 10:* Coordinates of selected alternative design space

The determination of an alternative design space for this problem statement is only enabled by applying the approach presented in this paper. Nevertheless, a lot of research work has to be done to enhance the determination of alternative spaces. Therefore, appropriate quality measures of the hypercuboids have to be introduced, the overlapping of different hypercuboids have to be checked automatically and the determination of best suitable hypercuboids has to be further improved.

## 6 Conclusions

In the presented approach, a new method is introduced, which enables a design update under reliability aspects in an early design stage. Therefore, a sequence of innovative methods is aligned to achieve the specified aim. The methods include a reliability assessment of the structure or process under investigation and a procedure to determine on the basis of those results alternative design spaces. Thereby, the presence of a possibly ambitious structural behavior is taken into consideration by means of cluster analysis to solve an inverse problem. Due to the consideration of an early design stage, the aim is not to detect an optimal, or best possible, design configuration but rather to eliminate unwanted regions from the design space.

The presented approach focuses on an application for industry-relevant problems. Here, the design of a deep drawing process is considered.

### Acknowledgements

The authors gratefully acknowledge the financial support of German Research Foundation (DFG) under the project SFB 528 TP7 'Adaptation and application of non-deterministic simulation and design strategies in the engineering practice'.

## 7 Literature

[1] J. C. Bezdek, R. Ehrlich, and W. E. Full. FCM: The fuzzy c-means clustering algorithm. *Computers & Geosciences*, 10:191–203, 1984.

[2] S. Ferson, C. A. Joslyn, J. C. Helton, W. L. Oberkampf, and K. Sentz. Summary from the epistemic uncertainty workshop: consensus amid diversity. *Reliability Engineering & System Safety*, 85(1–3):355–369, 2004.

[3] A. K. Jain. Data clustering: 50 years beyond k-means. *Pattern Recognition Letters*, 31:651–666, 2010.

[4] M. Liebscher. *Dimensionierung und Bewertung von Tragwerken bei Unschärfe – Lösung des inversen Problems mit Methode der explorativen Datenanalyse*. Dissertation (issue 17), TU Dresden, Fakultät Bauingenieurwesen, Institut für Statik und Dynamik der Tragwerke, 2007.

[5] B. Möller, M. Liebscher, S. Pannier, W. Graf, and J.-U. Sickert. An inverse solution of the lifetime-oriented design problem. *Structure and Infrastructure Engineering*, 2008.

[6] B. Möller and M. Beer. *Fuzzy Randomness – Uncertainty in Civil Engineering and Computational Mechanics*. Springer, Berlin, 2004.

[7] B. Möller and M. Beer. Engineering computation under uncertainty – capabilities of non-traditional models. *Special issue of Computers & Structures*, 86:1024–1041, 2008.

[8] R. E. Moore. *Interval Analysis*. Prentice Hall, 1966.

[9] Z. Qiu and I. Elishakoff. Anti-optimization technique – a generalization of interval analysis for non-probabilistic treatment of uncertainty. *Chaos, Solitons & Fractals*, 12:1747–1759, 2001.

[10] U. Reuter. Application of non-convex fuzzy variables to fuzzy structural analysis. In D. Dubois, A. A. Lubiano, H. Prade, M. Ángeles Gil, P. Grzegorzewski, and O. Hryniewicz, editors, *Soft Methods for Handling Variability and Imprecision*, volume 48, pages 369–375. Springer Berlin, 2010.

[11] L. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

 