Eigensolution Technology in LS-DYNA®

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LSTC

Nordic LS-DYNA Users' Conference 2016

What you are going to hear

- The variety of eigensolver technology in LSDYNA
- Coming Attractions
- Lots of Mathematics

Where do Eigenproblems come from

- LS-DYNA Mechanics solves the Conservation of Momentum Equation using a LaGrangian formulation
- Application of the FEM discretization yields a 2nd order system of Ordinary Differential Equations

$$M\ddot{u} + C\dot{u} + Ku = F$$

Characteristic Equation

- The Characteristic Equation approach is used with $u = \sum \alpha_j e^{i\omega_j t} \Theta_j$
- Then Presto Change-o you get

$$i^2 \omega_j^{\,2} M\Theta_j + i\omega_j C\Theta_j + K\Theta_j = 0$$

- With C = 0 you get

$$-\omega_j^{\,2} M\Theta_j + K\Theta_j = 0$$

$$K\Theta_j = \omega_j^{\,2} M\Theta_j$$

$$K\Theta = M\Theta\Lambda$$

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$$K\Theta = M\Theta\Lambda$$

- This is the standard eigenproblem in FEM
- LSDYNA uses Block Shift and Invert Lanczos in both SMP and MPP to solve this problem
- Lanczos requires one of K or M to be positive semidefinite.
- Standard Buckling Analysis uses Lanczos to solve

$$K\Theta = K_G\Theta\Lambda$$

Other Options

- For Model Analysis you can
  - Add dynamic terms to K to mimic the nonlinear iteration matrix
  - Solve just $K\Theta = \Theta\Lambda$
  - Thermal Conduction Matrix using *CONTROL_THERMAL_EIGENVALUE

Buckling with Inertia Relief

- Buckling with Inertia Relief is the first problem that does not meet the criteria for Lanczos.
- Inertia Relief constraints are imposed with LaGrange Approach and makes K indefinite.
- We added the Power Method to compute a small number of buckling modes.
- Power Method is not recommended for general use.

Quadratic and Unsymmetric

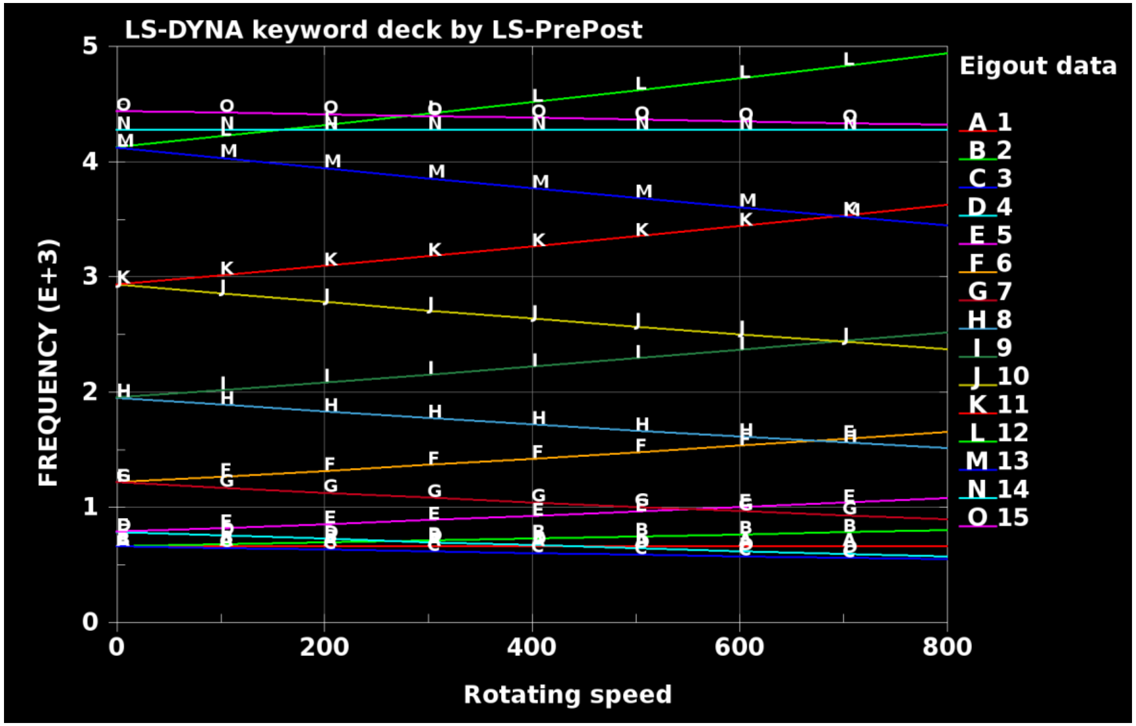
- Quadratic and Unsymmetric Eigenproblems arise by adding more physics
  - Unsymmetric material properties
  - Unsymmetric contact properties
  - Rotational Dynamics
  - First Order Damping terms

Examples

- Rotational Dynamics
- Brake Squeal

Campbell Diagrams

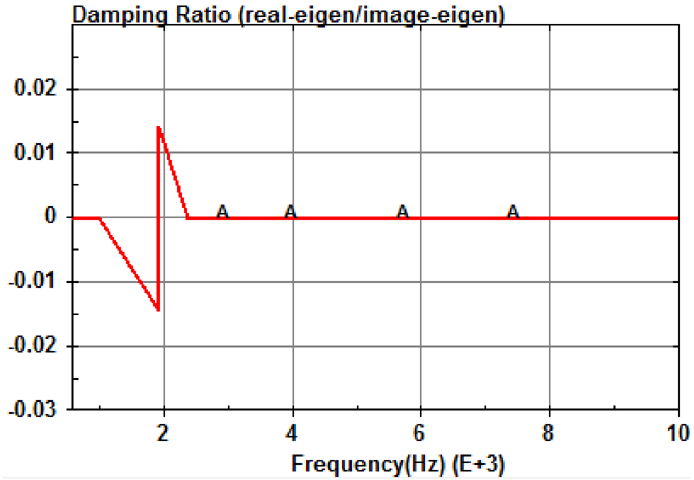
- Campbell Diagrams are plots of eigenvalues as a function of rotating speed
  - Need to track modes as they change

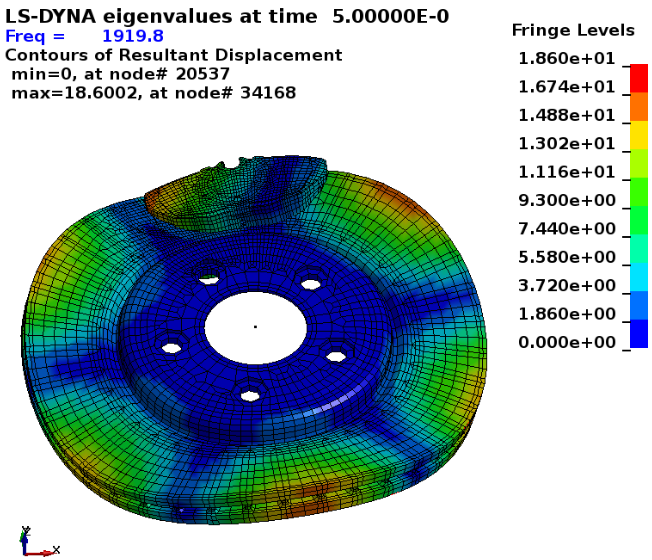


Unstable Mode for Brake Squeal

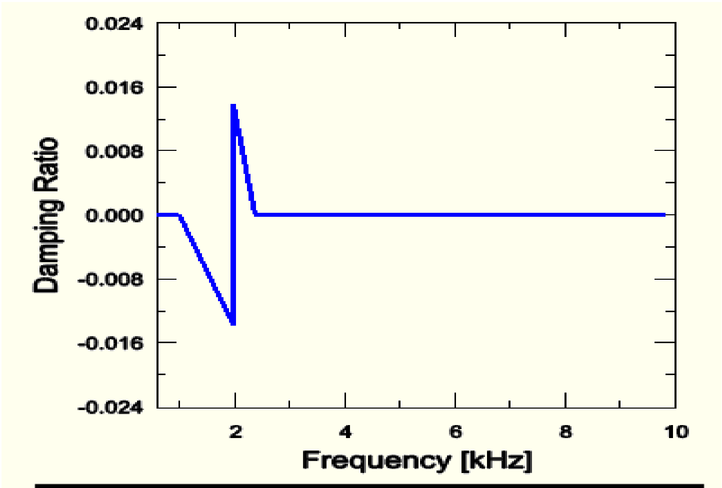
Damping Ratio is defined as -2*Re(λ) / |Im(λ)|, where λ is the eigenvalue.
When damping ratio is negative, unstable mode appears.

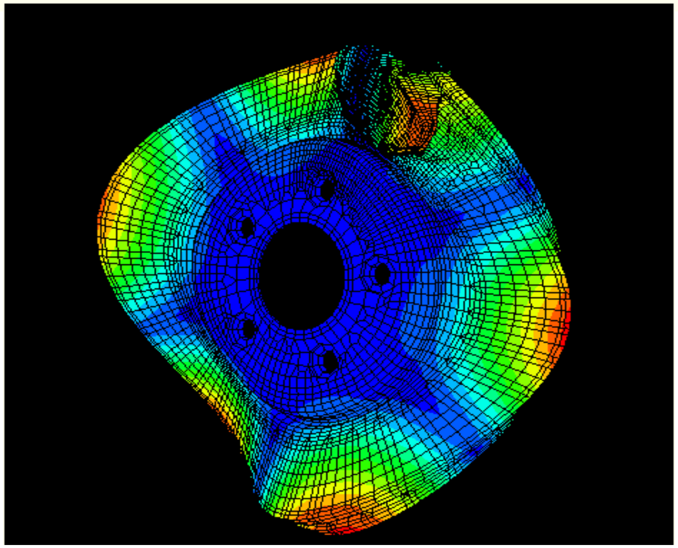
DYNA Result





Example Result





Back to Basics

- The Characteristic Equation approach is used with $u = \sum \alpha_j e^{\omega_j t} \Theta_j$
- No "i" (like a mathematician)
- Then Presto Change-o you get

$$\omega_j^2 M\Theta_j + \omega_j C\Theta_j + K\Theta_j = 0$$

- Eigenproblem stays Real!!!
- Eigenmodes are Complex

Conversion to First Order

- Quadratic Eigenvalue Problems have to be converted to First Order
- Use $\Psi_j = \omega_j \Theta_j$ to get

$$\omega_j M \Psi_j + C\Psi_j + K\Theta_j = 0$$

- Which becomes the First Order Eigenproblem

$$\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} \Theta \\ \Psi \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \Theta \\ \Psi \end{bmatrix} \Omega$$

Real Eigenproblem

- We use ARPACK
  - Public Domain eigensolver based on Arnoldi
  - Reverse Communication
  - At this time only SMP
  - MPP will be done in the next year
- Left hand side matrix has an easy inverse
- Requires factorization of real (symmetric or unsymmetric matrix) K
- Requires multiplications with C and M

Eigenmodes

- The matrices are real but the eigenmodes are complex
- D3EIGV database uses two states to hold the real part and then the imaginary part of the eigenmode.

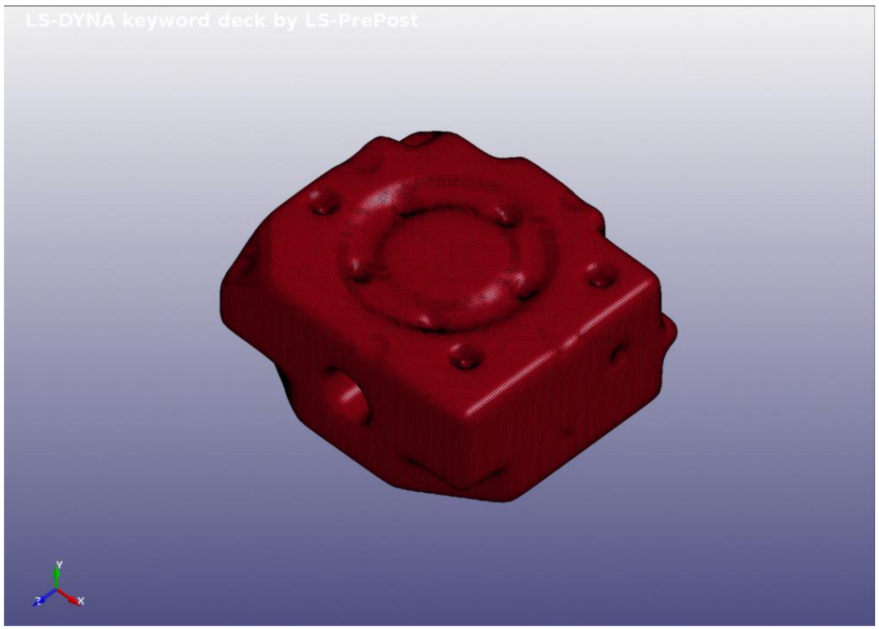
How do I use this?

- For the most part the decisions of when to use the new eigensolver features are made by LSDYNA
  - User controls Rotational Dynamics
  - User controls Symmetric or Unsymmetric
  - LSDYNA controls the eigensolver
- If user selects unsymmetric all damping terms in the model are included in the eigenvalue problem

MCMS

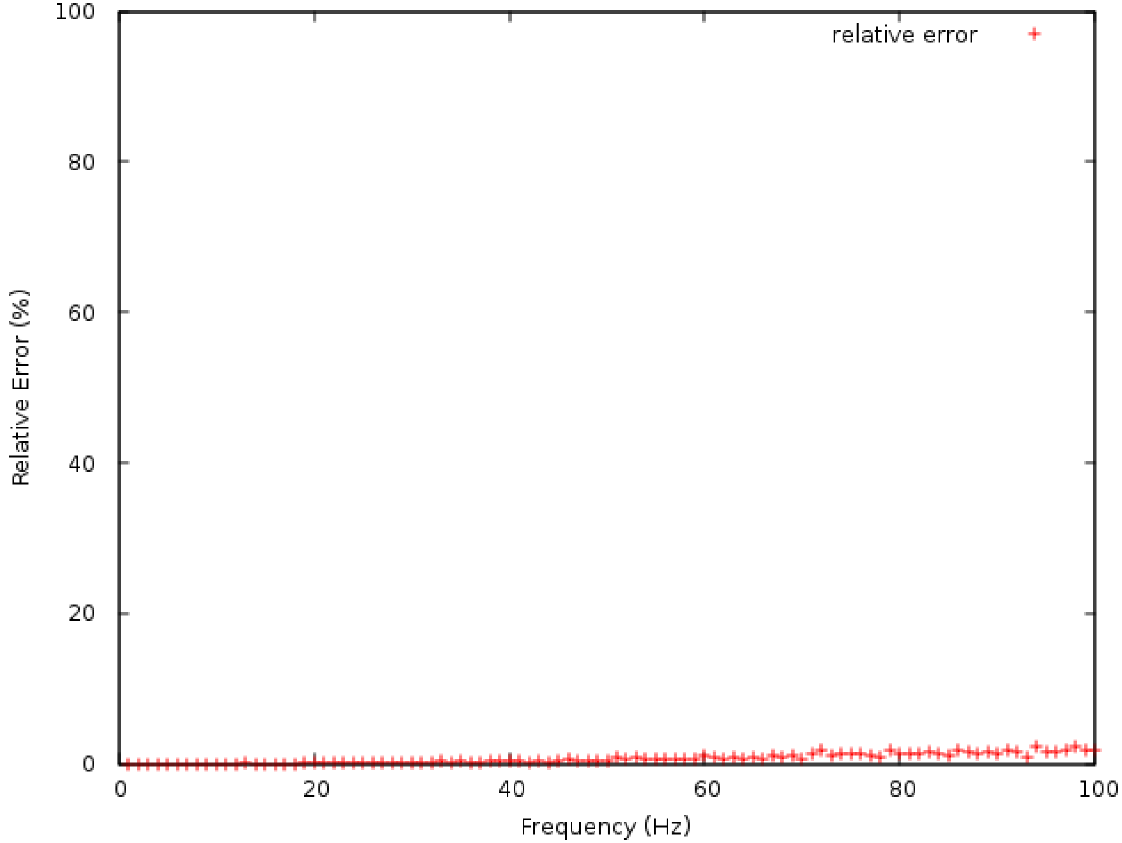
- LSTC is implementing the AMLS algorithm for computing approximate eigenmodes.
- Useful for applications that want thousands of modes for Frequency Response computations
  - Less accurate than Lanczos
  - But far less computer resources
- Noise, Vibration, and Harshness is the target application
- We are being assisted by Dr. Chang-wan Kim, School of Mechanical Engineering, Konkuk University, Korea
- We will be using the acronym of Multilevel Component Mode Synthesis or MCMS

Fuel Tank FE model

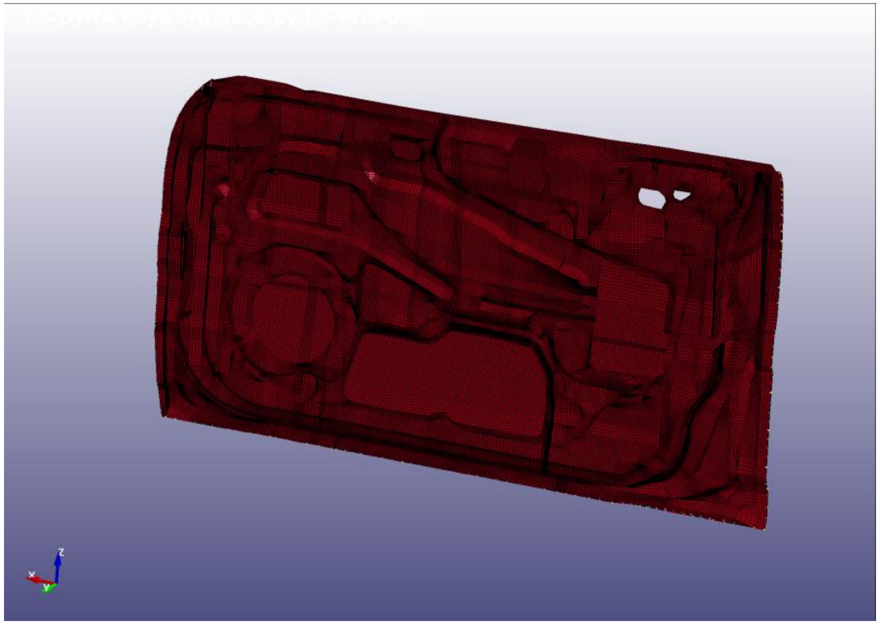


- 61,488 shell elements
- 323,832 DOF
- Normal mode analysis
  - Lanczos vs. MCMS
- Modal frequency response analysis (SSD)

Fuel Tank FE Model

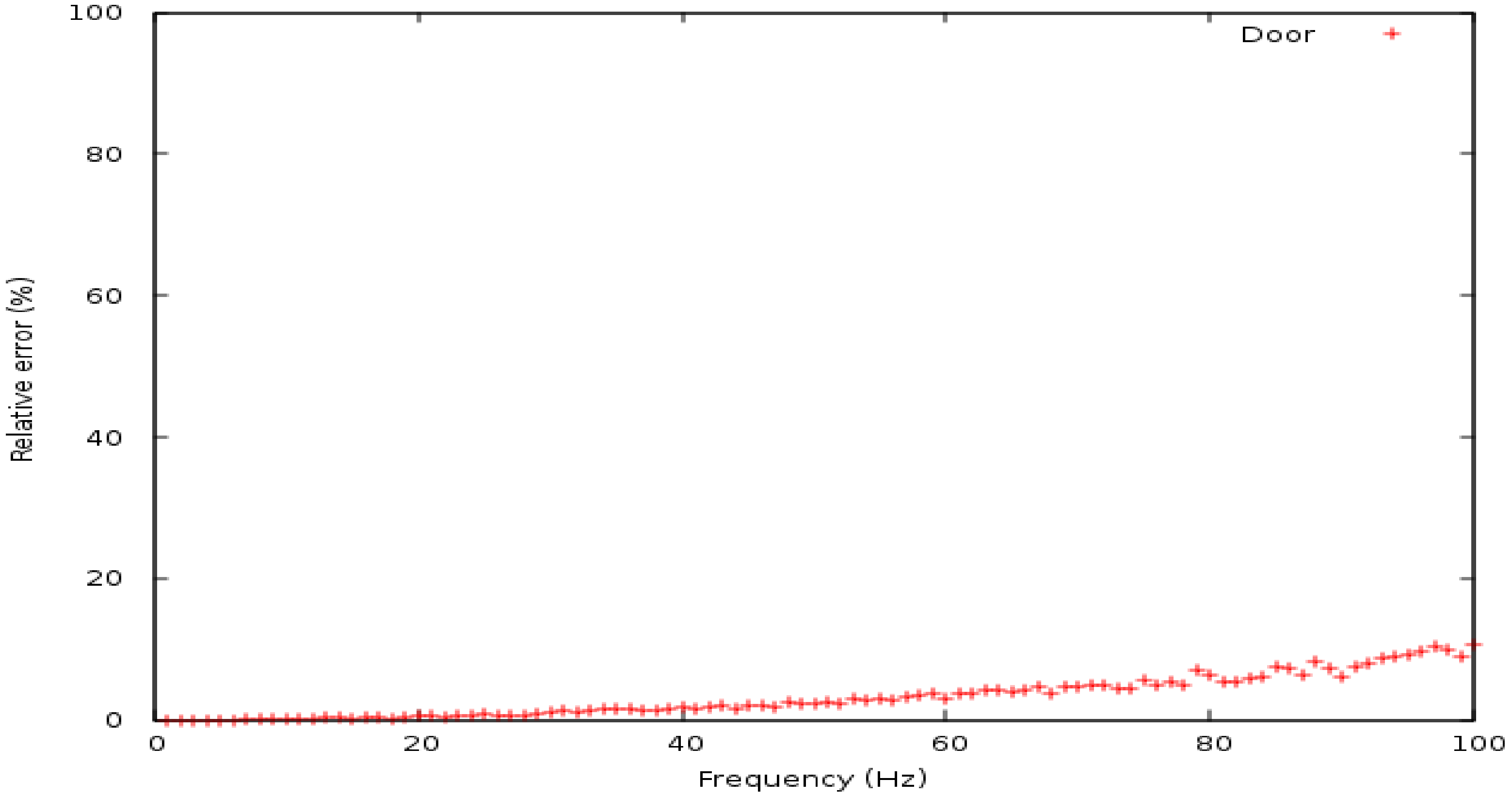


Door FE model

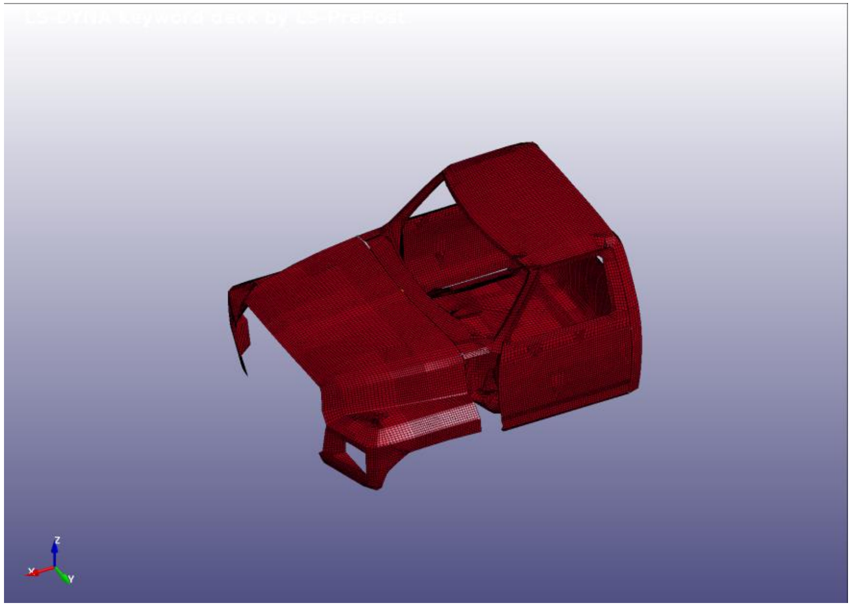


- 486,068 shell elements
- 2,915,562 DOF
- Normal mode analysis
  - Lanczos vs. MCMS
- Modal frequency response analysis (SSD)

Door FE model

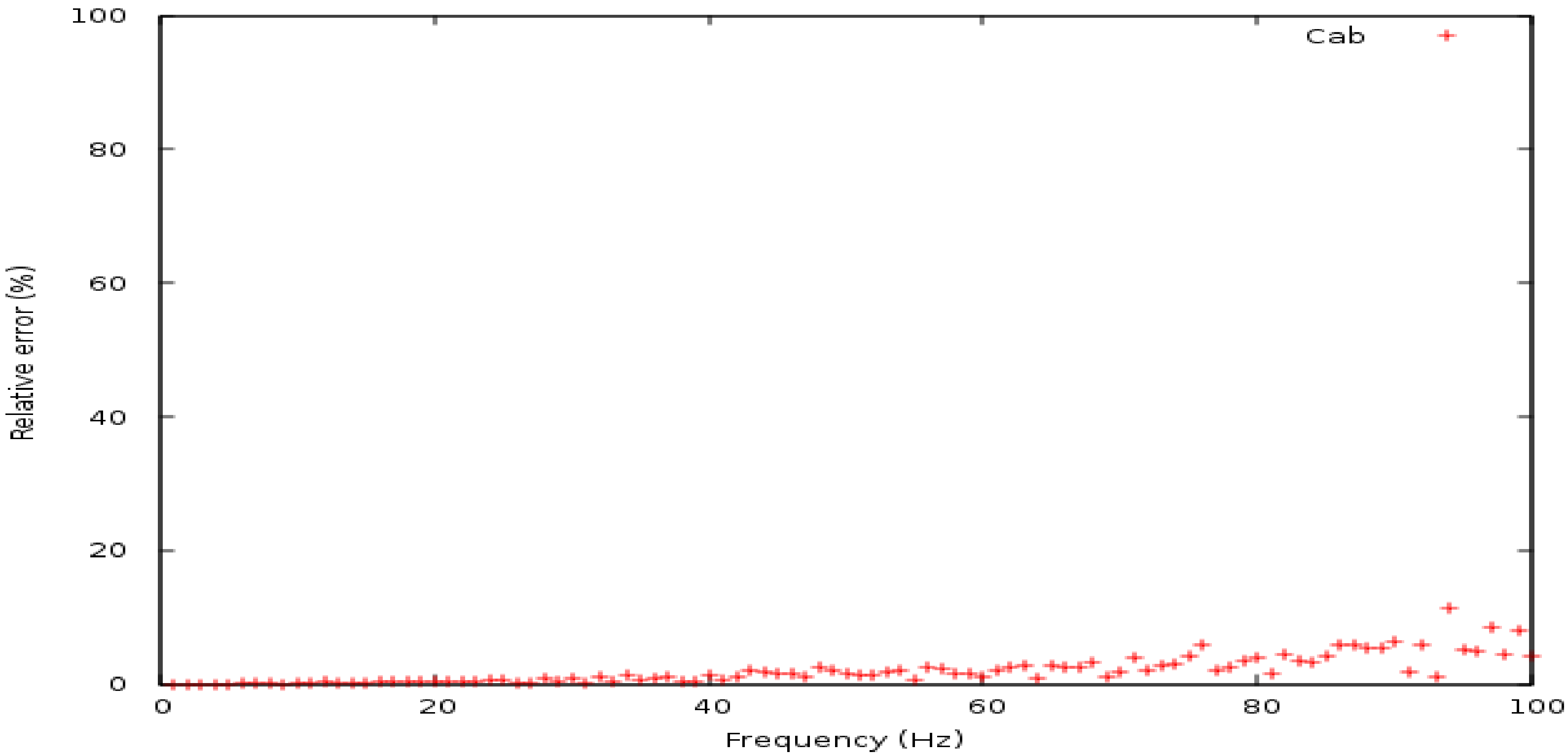


Truck Cab FE model

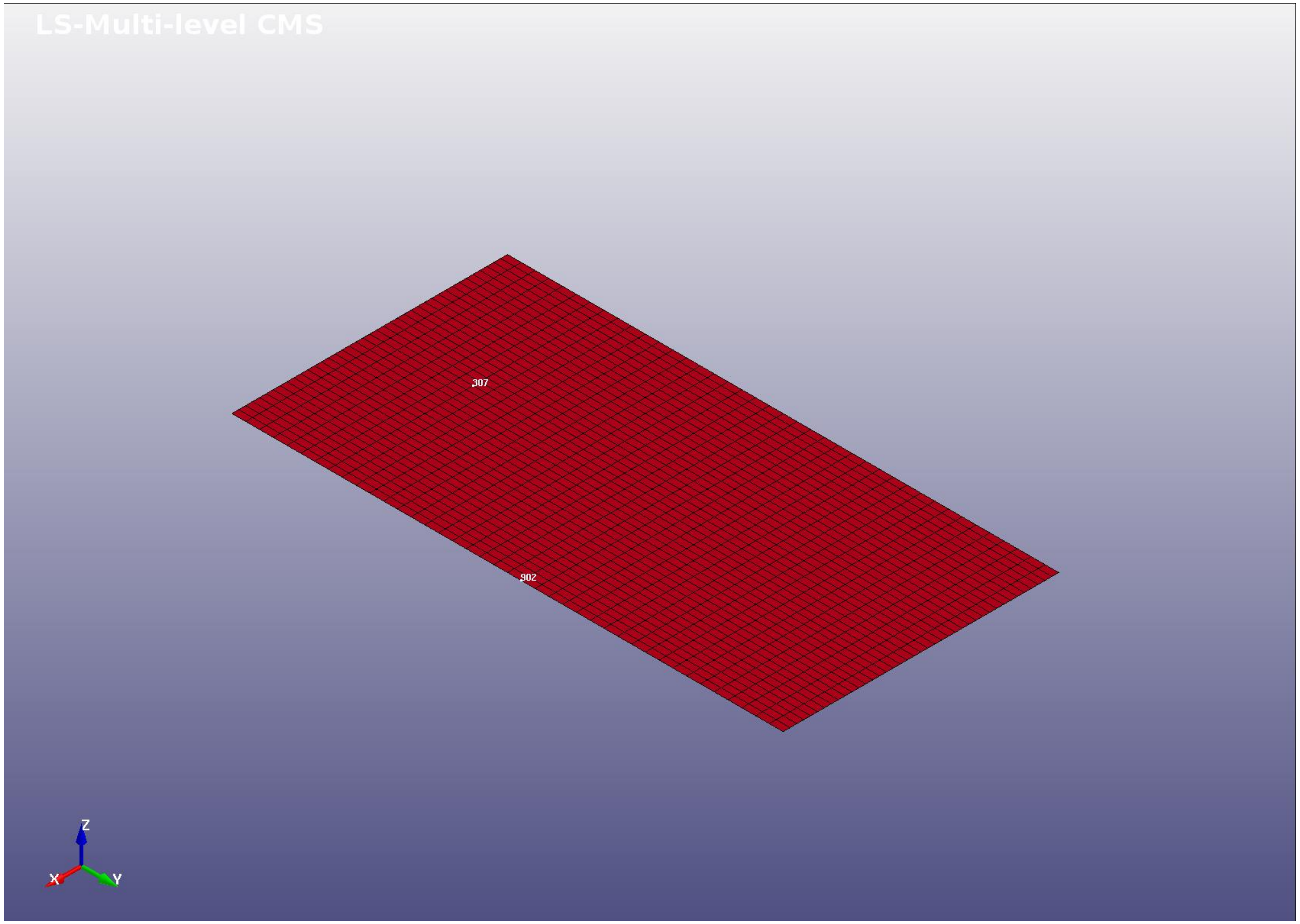


- 49,390 shell elements
- 296,274 DOF
- Normal mode analysis
  - Lanczos vs. MCMS
- Modal frequency response analysis (SSD)

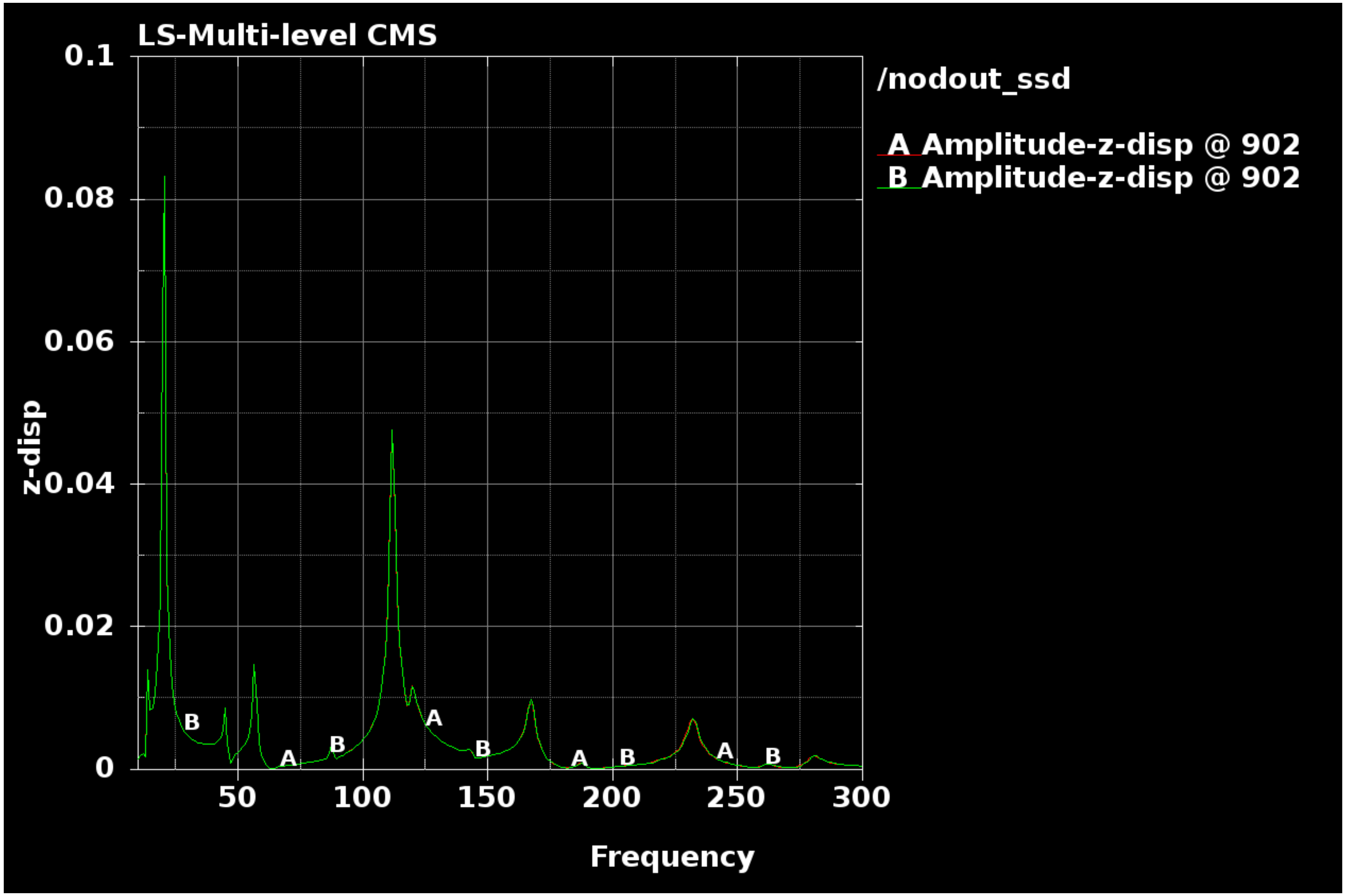
Truck Cab FE model



Simple Test Case



FRF Comparison



MCMS

- We have a serial implementation working
- But it still has issues to be resolved
- SMP version should be production ready sometime soon.

The End

- Thanks for listening.